Standard and non standard tests of alternative gravity theories

dr **Vincenzo Salzano**

Self-Report

Summary of professional achievements

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1 Personal Data

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2 Education and Degrees

Ph.D. in Fundamental and Applied Physics, 11.2005 - 12.2008	Ph.D. studies at the University of Naples "Federico II", completed obtaining a final Ph.D. mark of "Excellence". Ph.D. thesis title: *"Constraining Extended Theories of Gravity by Large Scale Structure and Cosmography"*. Supervisor: prof. Salvatore Capozziello.
Master Degree in Physics, 10.1999 - 05.2005	Master studies at the University of Salerno, completed with a final average exams mark of 29.3/30, and a final thesis discussion evaluated with the final mark of 110/110 *cum laude*. Master Thesis title: *"Evolution of hot stellar systems through the use of the fundamental plane of galaxies"*. Supervisor: prof. Salvatore Capozziello.

3 Professional Experience

Senior Postdoctoral Researcher 09.2014 - present	Institute of Physics, Faculty of Mathematics and Physics at the University of Szczecin.
Postdoctoral Researcher 02.2010 - 08.2014	Department of History of Science and Theoretical Physics, Faculty of Science and Technology at the University of the Basque Country, Leioa (Spain).
Young Postdoctoral Researcher 09.2009 - 01.2010	Institute for Theoretical Astrophysics, University of Oslo (Norway). Winner of a 10-months fellowship (reduced to 6 months to start a new job) under the mobility programme "YGGDRASIL - Young Guest and Doctoral Researchers", funded by the Research Council of Norway.

Vincenzo Salzano

4 Scientific achievement being the basis for the habilitation procedure

The scientific achievement, in accordance with the art. 16 paragraph 2 of the Act of March 14th, 2003, concerning the scientific degrees and titles (Dz. U. no. 65, item 595, as amended), is the series of publications entitled:

Standard and non standard tests of alternative gravity theories

In the following list I report the data regarding the publications entering the habilitation procedure, together with the description of my personal contribution to each of them. I have decided to include all my most recent papers, which I have been able to develop since I have been appointed as Postdoctoral Researcher in Poland, in the Cosmology Group of the Institute of Physics at the University of Szczecin, in order to show the high degree of collaboration I have reached with all the members of my present group, and to enlighten the possibility of a very fruitful future collaboration. For each publication, I provide the Impact Factor by year of publication derived from Journal Citation Report (JCR).

SA1. Balcerzak A., Dąbrowski M. P., **Salzano V.**, *"Modelling spatial variations of the speed of light"*, Annalen der Physik 529 (2017) no.9, 1600409.
DOI: 10.1002/andp.201600409.

In this work we show how a varying speed of light can mimic an inhomogeneity signal on cosmological scales, and how it is possible to detect and disentangle both of them by using some cosmological probes. My contribution consisted in: performing part of the numerical calculations, using a method developed by myself in two previous papers, Ref. [**SA5**,**SA6**]; discussing the results obtained; and giving an active contribution to the article writing.

My percentage contribution is estimated at about 33%.
Impact Factor: 3.443 (2016 JCR).

SA2. **Salzano V.**, *"Recovering a redshift-extended varying speed of light signal from galaxy surveys"*, Phys.Rev. D95 (2017) no.8, 084035.
DOI: 10.1103/PhysRevD.95.084035.

In this article I show how it will be possible to extract a varying speed of light signal from future galactic surveys, by improving and extending a method which I proposed in two past works, Ref. [**SA5**,**SA6**]. My contribution consisted in: performing all the required numerical calculations; discussing the method and the results; and writing the article.

My percentage contribution is estimated to be 100%.
Impact Factor: 4.506 (2016 JCR).

SA3. **Salzano V.**, Mota D.F., Capozziello S., Donahue M., *"Breaking the Vainshtein screening in clusters of galaxies"*, Phys.Rev. D95 (2017) no.4, 044038.
DOI: 10.1103/PhysRevD.95.044038.

In this article we show that it is possible to describe the internal dynamics of clusters of galaxies in a theoretical context alternative to general relativity, where no dark energy is present at cosmological scales but a new field with some well-defined properties. My contribution consisted in: performing all the required numerical calculations; discussing the method and the results; and writing most of the article.

My percentage contribution is estimated at about 75%.
Impact Factor: 4.506 (2016 JCR).

SA4. **Salzano V.**, Mota D.F., Dąbrowski M. P., Capozziello S., *"No need for dark matter in galaxy clusters within Galileon theory"*, JCAP 1610 (2016) no.10, 033.
DOI: 10.1088/1475-7516/2016/10/033.

In this paper we show that it is possible to describe the internal dynamics of clusters of galaxies in a theoretical context alternative to general relativity, where a new scalar field is introduced and which can play both the role of dark matter and dark energy, provided it has some suitable screening properties. My contribution consisted in: performing all the required numerical calculations; discussing the method and the results; and writing most of the article.

My percentage contribution is estimated at about 70%.
Impact Factor: 5.634 (2016 JCR).

SA5. **Salzano V.**, Dąbrowski M. P., Lazkoz R., *"Probing the constancy of the speed of light with future galaxy survey: The case of SKA and Euclid"*, Physical Review D93 (2016) no.6, 063521.
DOI: 10.1103/PhysRevD.93.063521.

This article comes as an update and an extension of a previous work, Ref. [**SA6**]: we show how it is possible to use future galaxy surveys to measure the speed of light on cosmological scales and, thus, test if varying speed of light theories are supported by data or not. My contribution consisted in: performing all the required numerical calculations; discussing the method and the results; and writing most of the article.

My percentage contribution is estimated at about 70%.
Impact Factor: 4.506 (2016 JCR).

SA6. **Salzano V.**, Dąbrowski M. P., Lazkoz R., *"Measuring the speed of light with Baryon Acoustic Oscillations"*, Physical Review Letters 114 (2015) no.10, 101304.
DOI: 10.1103/PhysRevLett.114.101304.

In this article we propose for the first time a new method which makes it possible to use future galaxy surveys to measure the speed of light on cosmological scales. In particular, such a method will give the possibility to check if the speed of light has varied in the past or not. My contribution consisted in: performing all the required numerical calculations; discussing the method and the results; and writing most of the article.

My percentage contribution is estimated at about 70%.
Impact Factor: 7.645 (2015 JCR).

4.1 Description of the academic achievement

4.1.1 Introduction

All my present and near future research is focussed on a well-defined topic, but with multiple interests and applications: to study the feasibility of alternative theories of gravity as reliable substitute candidates to general relativity (GR), both at cosmological and astrophysical scales. I find extremely fascinating the possibility that the ultimate theory of gravity is not GR but one more general for which, among others, dark energy and dark matter are not physical energy-matter fluids, but manifestations of a breakdown of our understanding of the physics of our universe. And it is even more exciting to imagine that such breakdown might lead to reveal some - up to now - unknown new aspect of physics which might also have influence on our daily life.

My main research line, at the moment, is strictly connected to the project I am funded by, and in line with such expectations: to explore the possibility to detect observational signatures of a space-time variation of the fundamental constants of Nature or, better, of our "current understanding of the Nature". This is a quite largely debated topic, but I am facing it as a way to change perspective: sometimes it happens that one gets stuck and needs to change his own point of view in order to understand better the problem he is struggling with, and to make remarkable progresses in his own work. In this sense, in my very personal opinion, even *too-much-alternative* approaches (or, at least, considered as such by part of the scientific community) can be useful, also to deepen the understanding of more standard paths.

While the variation of some quantities like the fine structure constant has been explored in literature in some detail, both from the theoretical [1, 2, 3] and the observational [4, 5, 6, 7] side, I have decided to focus my attention on a much more tricky quantity: the speed of light, c. At some level, this is an even more fundamental quantity with respect to other constants of Nature, due to the impact it has on many branches of our physics [8] and maybe, just for this reason, its omnipresence and our over-familiarity with it, it looks more off-limit than others quantities and any attempt to questioning its status is somehow considered too much heretical. As better explained in the next pages, I think that the so-called Varying Speed of Light (VSL) theories deserve more attention than they are paid of nowadays, due to the profound insights they might bring to our knowledge of the universe. They are not free of problems, of course, but we do not have to forget that even the currently most accepted *standard* cosmological model, the so called ΛCDM (cosmological constant + cold dark matter) is not perfect and completely satisfactory, if the scientific community feels the need to extend or modify it, in order to address some of the problems which observations pose to it and to us [9].

In order to face this topic in a proper way, I think that to carry a phenomenological approach out, as it is done in many cases, with dark energy for example, namely, to choose some ansatz to parameterize a possible variation of c, and to test it against observations, is very far from being of any help. This would not be a direct measurement of c, and would be biased by our choices, resulting quite useless at this stage. I am more interested in finding direct measurements or other direct signatures, if possible, maybe through stellar evolution processes, large scale structure, or other cosmological and astrophysical probes to be identified.

If the main goal of cosmology now and in the near future is *to test* GR, then, why not to try all the possible alternatives and deviations? I personally think that to keep proceeding in the way we are doing now, is fated to crash. Statistically speaking, the ΛCDM model will always result to be the best model, for its intrinsically statistical simpleness. And I also think that present cosmological data are almost fully saturated (at least the geometrical probes, while the dynamical one are still at a low-accuracy level to be considered as decisive) for what concerns a well-settled selection among different dark energy models and/or parameterizations (here I use the term "models" if derived from theory, and "parameterizations" if from phenomenological proposals); and are even "only slightly more-than-useless" when it comes to discriminate among GR and alternative theories of gravity. If we really want to confirm or confute a given scenario or approach, we have to search for the (in)famous "smoking gun(s)". And what better way to find them than to move away from the well-known road, in favour of largely unexplored and unexpected new paths? Following my latest research, I came out even more convinced that we have to dare, now more than in the past, because the technological progress we are and will be experiencing, might result not to be enough to deepen our understanding of the universe, if it is not accompanied by a change in our scientific minds and perspectives, even taking the risk of trying alternative paths. Being *"dwarfs standing on the shoulders of giants"* may not be enough, if we *"don't dare to look beyond and farther"*.

4.1.2 Varying speed of light theories vs cosmological observations: preamble

The article reported in the above list as Ref. [**SA6**] has been the first work we have performed about the observational detection of varying fundamental constants. We have focused on the out-of-mainstream set of theories, the so-called Varying Speed of Light theories. There is a large debate about the soundness and even the need of such theoretical constructions. The main criticism is that the speed of light, c, is a dimensional quantity and, as such, any investigation about its variation is misleading and not well-based, because one can always build a unique set of units of length and time for which c is constant. In other words, dimensions might be changing, not the speed of light itself. That is why, for some scientists, the only reasonable fundamental constants which should be investigated in this context are dimensionless quantities like, for example, the fine structure constant. We personally don't agree entirely with these caveats for many reasons.

First: if we start from the beginning, i.e. from the Lagrangian, we can introduce the speed of light as a new scalar field, and take it into account properly in the derivation of all the cosmologically useful equations. And the fields always have units (dimensions). This is a very general approach for all dimensional quantities. Moreover, the discussion about the dimensionality of c should also refer to the dimensionality of G_N, the gravitational constant, which is rarely objected by physicists dealing with Brans-Dicke theory [10]. A caveat about some VSL theories, is that a derivation from first principles is not always performed; and we agree this is a problem.

Second: the variation of c, or of any dimensional quantity, can be always related to the variation of a dimensionless quantity as, for example, in this case, the fine structure constant α. Varying fine structure constant theories have been studied intensively both in the theory [1, 2, 3] and in observations [4, 5, 6, 7] in the last two decades, and apparently they all started from the Bekenstein theory of varying electron charge e model [11], which seems to be on the same footing as VSL models, being e a dimensional quantity. Both e and c can be related to the fine structure constant, through its definition, $\alpha = e^2/\hbar c$, where $\hbar$ is the Planck constant. Thus, a varying α might also correspond to a varying c (or e).

Third: if it is true that we can define a unit system where c is constant *by definition*, as it happens in the nowadays accepted International System (SI), it is also true that we can fix a new system of units in which c can be safely considered varying. There is no unique way to introduce a VSL theory, basically because different choices of units can lead to different varying quantities but the same effective theory. But if we follow the approach from [12, 13, 14], which is based on the assumption that the quantity $Q \equiv \hbar/c$ is constant, together with the electron mass, m_e, and the electron charge, e, we can easily derive the corresponding unique set of constant units of mass (M), length (L) and time (T) [15]:

$$u_M = m_e\,, \qquad u_L = \frac{Q}{m_e}\,, \qquad u_T = \frac{Q^{3/2}}{m_e e'}\,, \tag{1}$$

where $e' = e/\sqrt{4\pi\epsilon_0}$, with ϵ_0 the vacuum permittivity. In such a system c can be made healthily varying; we can call this new system as the VSL-US (Varying Speed of Light Unit System), which we can now compare to the standard SI. If we introduce (and will define later) the concepts of *fundamental units* and *experimental units*, we can see that in the VSL-US we can properly define the fundamental units so that c can be safely considered as a varying quantity, while the

VSL-US experimental units are completely equivalent to the SI ones. Note that the newly defined fundamental units u_X, in terms of the *experimental units* of the SI, would correspond to:

$$u_M \sim 10^{-31}\, kg\,, \qquad u_L \sim 10^{-13}\, m\,, \qquad u_T \sim 10^{-20}\, s\,. \tag{2}$$

At least in principle, one could re-define the experimental SI units (kg, m and s) in terms of the new VSL-US fundamental ones: we can still go on using, for example, the meter as a length unit, but, in the VSL-US, the meter is no more defined through the second and the (constant) speed of light, but in terms of u_L. It should be clear now why we have chosen to describe units as fundamental and experimental: the definition of the fundamental units can be completely free and dictated by theoretical requirements; experimental units are to be defined responding to the reasonable criteria of *reproducibility* and *practicality*.

At this point, it is not useless to show that such way to proceed is by no means new, but quite customary, even though mostly implicit. Let us consider the second, as it is defined now in the SI: from the same official page of the Bureau International des Poids et Mesures (BIPM), the second *"is the duration of* 9, 192, 631, 770 *periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium* 133 *atom."*[1]. Such duration was officially defined in 1967/68, by matching atomic clock measurement (from microscopic scales) with the ephemeris second (from macroscopic scales). In this case, the frequency of the atoms is the *fundamental unit*; the ephemeris second is the *experimental unit*. Of course, there was an improvement in that the newly define "time stick/ruler", i.e. the atomic clock, was much more stable than the previous astronomical definition. But what has to be taken home from this discussion, is that the second, as time interval, was the same before and after this definition was made official. The same reasoning applies to the meter. Since 1983 the meter is officially defined as *"the length of the path travelled by light in vacuum during a time interval of* 1/299, 792, 458 *of a second."*[2]. Thus, as said above, in the SI, we need to fix the second and assume that the speed of light is constant. But in the VSL-US, we can simply state that the meter is some multiple of the *fundamental unit*, u_L. At first sight, one might claim that such unit violates any reasonable criterion of *reproducibility* and *practicality* which we have invoked above. But, as we have said, at least theoretically, this new system of units sounds correct and, within it, the assumption of a varying speed of light is correct.

Another consequence of such reasoning is that the speed of light, *here and now*, can be still considered *numerically* equal to the value 299792.458 km/s that it has in SI[3]. The difference is in the implicit definition of the *fundamental units*, but not in the *experimental ones*, the meter and the second, which are unchanged. Note also that after 1973 and before 1983, the value of the speed of light was measured as *"the product of the frequency and wavelength of an electromagnetic wave is the speed of propagation of that wave."* [16]. Avoiding technicalities, what was practically done was to measure the frequency (proportional to time) and the wavelength (a length) of a well-defined laser, and from them the value of the speed of light in vacuum was derived. Thus, a velocity was derived from a ruler and a clock. In order to do that, of course, you needed to define the *experimental units* of the second and of the meter.

Furthermore, we cannot avoid to underline here that even in the standard case where c is constant (SI units) the cosmological distances (which are the main quantities we are going to work with) are calculated indirectly by parallax (from which the unit of the *parsec* is defined) given that there would be no ruler working properly both locally in our laboratories and at such large scales. Moreover, cosmological distances are calculated theoretically by multiplying the speed

[1]http://www.bipm.org/en/publications/si-brochure/second.html
[2]http://www.bipm.org/en/CGPM/db/17/1/
[3]http://www.bipm.org/en/CGPM/db/15/2/; http://www.bipm.org/en/CGPM/db/17/1/

of light with some integral of the scale factor. Then, using the conversion factor between the parsec and the meter, which is intrinsic to the definition of the former, one can find the numerical value of cosmological distances in units of parsec. The main assumption underlying cosmological-scale measurement processes is that units (once chosen) are invariant in time and space; if not, then, only local physics would be correct, while all the entire cosmology would be based on fallacies. In the c-constant scenario, the meter is fixed by c itself; in the VSL-US we have a new *fundamental unit* of length, but always the same *experimental unit*. Thus, we are only assuming that the conversion is between the parsec and the meter, with the latter being defined with respect to new fundamental units.

Fourth: for what concerns a varying c, we cannot avoid to point out a "reality question". VSL theories were originally introduced in order give an alternative scenario to inflation [17, 18]: an accelerated expansion of the universe might be mimicked, actually, by a faster speed of light. And it could work not only at inflationary epochs, but also for dark energy dominated eras. Now, the accelerated expansion is a real measurable phenomenon, with real physical consequences; could it be reduced to a simple matter of units? Actually, from the VSL perspective, this strongly depends on the way the VSL theory is built up: As pointed out in [15], if the theory is covariant and Lorentz invariant (as GR is), then, one can always find a choice of time unit in which the VSL theory is identical to the standard cosmological model, namely, such that $c(t)\,dt = c_0\,dt'$. But if a theory violates one of the previous conditions, or both, as it happens, for example, with models developed in [12, 13, 19], then a preferred frame exists where it does make sense to talk about a physical VSL signal.

Finally, we could conclude saying that, luckily, Nature is absolutely indifferent to our arbitrariness in choosing a unit system, or to our present level of understanding of the physics leading its phenomena.

4.1.3 Varying speed of light theories vs cosmological observations: the method

Anyway, adopting a conservative approach, in our work, Ref. [**SA6**], we have found that a constant speed of light, or a more general VSL theory, is intimately related to a newly-defined dimensionless parameter, which is strictly equal to 1, if the speed of light is constant, and different from 1, if the speed of light is varying.

The most interesting point is that such parameter can be easily measured while a galaxy survey is running. One of the main targets in a galaxy surveys is the measurement of the clustering or correlation function among those galaxies. In their clustering, a typical correlation length is imprinted, the evolved sound horizon, strongly related to the Baryon Acoustic Oscillations (BAO) [20, 21]. In a BAO survey, such correlation length can be measured in two different directions: in a transversal one, along the plane of the sky, where it is seen subtending a certain angle and, as such, we relate it to an angular diameter distance; and in a radial or longitudinal direction, which we derive from the spatial and redshift galaxy distribution along the line of sight. These two BAO modes, as they are called, are defined as:

$$y_t = \frac{D_A}{r_s} \quad ; \quad y_r = \frac{c_0}{H\ r_s}\,, \tag{3}$$

where c_0 is the value of the speed of light here and now, D_A is the angular diameter distance, H is the expansion rate of the universe, and r_s is the sound horizon. We start from the definition of the angular diameter distance which, assuming spatial flatness, is:

$$D_A(z) = \frac{1}{1+z}\int_0^z \frac{c(z')}{H(z')}dz', \tag{4}$$

where $H(z)$ is the Hubble function (expansion rate); and $c(z)$ is the speed of light expressed as any possible function of redshift. In a standard scenario, the speed of light is constant and $c(z) = c_0$; in the more extended context of a VSL approach, it can be any function, unknown to us until we do not recover it from the data. For what concerns $H(z)$, instead, in principle it can be derived from the first Friedmann equation, in combination with a continuity equation, once a cosmological theory is given. It can be easily checked that the angular diameter distance has a peculiar property: it is small (tending to zero) for closer objects; grows as we proceed farther from us (i.e. grows with redshift); reaches a maximum and then starts to decrease. Thus, objects located at redshifts higher that a certain "maximum redshift" (defined as the redshift where D_A reaches its maximum) appear to be bigger than objects located at smaller redshifts. While this could sound weird, it is a consequence of a combination of many aspects (metric definition, spatial curvature) [22]. The crucial point is given by the condition satisfied by this "maximum redshift":

$$\frac{\partial D_A(z)}{\partial z} = 0 \quad \Rightarrow \quad D_A(z_M) = \frac{c(z_M)}{H(z_M)} \quad \Rightarrow \quad D_A(z_M)H(z_M) = c(z_M)\,. \tag{5}$$

Thus, if we have at our disposal independent measurements of D_A and H at the maximum redshift we can, in principle, measure the value of the speed of light at the maximum redshift. We could be able to prove or not the constancy of the speed of light on scales much larger than our Earth-based laboratories and at times much older than any kind of human experiment could have ever been conceived.

Unfortunately, BAO-only measurements are not able to provide us with such opportunity. It can be easily seen that the condition at the maximum always gives:

$$y_t(z_M)y_r^{-1}(z_M) = 1 \tag{6}$$

when expressed in terms of BAO modes[4]. That is because the radial mode is always expressed as c/H, with no possibility to distinguish between c or c_0, as long as one does not provide an independent measurement for H.

But BAO are fundamental to calculate the maximum redshift z_M. In fact, the profile of D_A about z_M is very flat over a large redshift range; observations are not continuous variables; and they are plagued by observational errors and dispersion. But the previous condition, Eq. (6), clearly sets that the maximum is where both the tangential and the radial modes are equal. A further additional problem is that on-going BAO surveys have not enough accuracy to measure tangential and radial BAO modes independently; thus, we are forced to rely on mock data derived from future BAO surveys prescriptions. In particular, in Ref. [**SA6**] we have focussed on the Square Kilometer Array[5] (SKA), and *Euclid*[6]. Even in this case, anyway, we need to build up an algorithm in order to find z_M. While more details can be found in Refs. [**SA5**,**SA6**], we report here only the main steps:

1. we need to set up a fiducial cosmological background: we start from the baseline ΛCDM model from *Planck* 2015 release, `base_plikHM_TTTEEE_lowTEB_lensing_post_BAO`, fully characterized by one cosmological parameter, the dimensionless matter density today, $\Omega_m = 0.31$, once spatial flatness is assumed. Upon this we need to add a possible VSL signal: given that

[4] We need the sound horizon to express D_A and H as y_t and y_r. It can be derived in an independent way from other cosmological probes as, for example, the Cosmic Microwave Background (CMB). See latest results from *Planck* telescope, [23].

[5] `https://www.skatelescope.org/`.

[6] `http://sci.esa.int/euclid/`.

we do not have any fundamentally based VSL functional form, we have chosen to work with an ansatz proposed in [24], i.e.

$$c(a) \propto c_0 \left(1 + a/a_c\right)^n \ , \tag{7}$$

where $a \equiv 1/(1+z)$ is the scale factor, and a_c is the transition epoch from some $c(a) \neq c_0$ (at early times) to $c(a) \rightarrow c_0$ (at late times - now). Then, we consider two different VSL signals: $\Delta c/c_0 \sim 0.1\%$ at $z_M \approx 1.55 - 1.6$, given by the baseline ΛCDM model plus a $c(a)$ given by Eq. (7), with $a = 0.05$, $n = -0.001$; and $\Delta c/c_0 \sim 1\%$ at $z_M \approx 1.55 - 1.6$ given by the baseline ΛCDM model plus a $c(a)$ given by Eq. (7), with $a = 0.05$, $n = -0.01$. The output of such assumptions will be a set of fiducial y_t^{fid} and y_r^{fid};

2. we assume that the errors on the observational quantities we need, i.e. y_t and y_r, are as described in literature for both *Euclid* [25] and SKA [26]. The errors so calculated are also useful because we will not work directly with the fiducial model values, y_t^{fid} and y_r^{fid}. Instead, we will randomly pick up values of y_t^{fid} and y_r^{fid} from a multivariate Gaussian centered on the fiducial values, and with a total covariance matrix built up from the errors we derived in the way previously described, and assuming an additional correlation factor between them, equal to $r \sim 0.4$, as derived in [27]. Such procedure is needed in order to give to mock data an intrinsic dispersion closer to the real one;

3. starting from the scattered mock data, we employ a highly model-independent reconstruction method in order to reconstruct y_t and y_r as almost-continuous functions. We have chosen the Gaussian Process algorithm [28] for this reconstruction process, using a very fine redshift grid, much more fine than the binned data from the surveys, in order to minimize numerical errors when determining z_M;

4. we find z_M by solving Eq. (6);

5. in order to avoid possible cosmic variance problems and the uncertainty about the assumed fiducial cosmological model, we repeat steps from 1 to 4 many times ($\mathcal{N} \sim 10^3$), so that we end up with $\mathcal{N}$ sets of (y_t, y_r) all over the redshift range covered by the given surveys and, for each of them, we find numerically the corresponding z_M;

6. eventually, we end up with a distribution of $\mathcal{N}$ z_M values; from this ensemble, we extract the median (which mainly depends on the used fiducial cosmological model) and the corresponding error (which mainly depends on the accuracy and precision of the surveys). This will define the possible detection of the maximum redshift for both the surveys.

Before to proceed, we need to clarify some things about point 1. If we want to implement a VSL signal in the cosmological distances, we need a VSL theoretical background. In Refs.[**SA5**,**S6**] we were not interested in testing which VSL approach was the best, or how well it could fit the data; we only needed to introduce a VSL signal in the observations, so that the choice of the theory was not important. Anyway, we needed to make a choice, in order to have a proper distance calculation. Following [12], the first Friedmann equation will look like:

$$H^2(t) = \frac{8\pi G}{3}\rho(t) - \frac{k}{a^2(t)}c^2(t) \ , \tag{8}$$

while the continuity equation is:

$$\dot{\rho}(t) + 3H(t)\left(\rho(t) + \frac{p(t)}{c^2(t)}\right) = \frac{3k}{4\pi G a^2(t)}c(t)\dot{c}(t) \ , \tag{9}$$

where: ρ and p are, respectively, the energy-mass density and the pressure of any fluid in the Universe; $a(t)$ is the scale factor; G is the universal gravitational constant; and the speed of light is expressed as a general function of time (or redshift), $c(t)$. What is interesting to note, is that any change produced by a VSL is connected with the spatial curvature. Thus, in our case, where we are working assuming the condition of spatial flatness, e.g. $k = 0$, this implies that no effective change is working in the continuity equation and, consequently, in the first Friedmann equation which, we underline, is directly connected to the observable quantity $H(z)$. But this is not the case for other VSL approached, like in [15, 19].

Moreover, in order to make the global dynamics of the Universe within the two VSL scenarios we have chosen compatible with present data, we have had to change the value of Ω_m, the dimensionless matter density today. This is expected because a VSL signal can mimic the effects of a dark energy fluid, i.e., an accelerated expansion; actually, VSL were introduced for the first time in [12, 29] as an alternative to inflation, namely, a faster speed of light in the past could mimic the fast acceleration of the universe at the base of the inflationary paradigm. A higher speed of light in the past can mimic the effects of a dark energy component, thus resulting in a lower value for Ω_{DE} (dimensionless dark energy density today). When no spatial curvature is assumed, this gets converted to a larger value of Ω_m. In order to arrange for the above assessed variations in c, in the first model of VSL, we need $\Omega_m = 0.314$, and in the second we need $\Omega_m = 0.348$. We stress anyway that such values are not derived from a fitting procedure to present cosmological data, which was out of the purpose of the work we are discussing here. We simply checked heuristically the values which could give a qualitatively good global description of present data.

Finally, we want to explain why we have chosen as reference value for the maximum redshift z_M of the VSL signal the interval $1.55 - 1.6$. We have considered the CPL [30, 31] $w + w_a$ `plikHM_TTTEEE_lowTEB_BAO_post_lensing` and the baseline (ΛCDM) model `plikHM_TTTEEE_lowTEB_lensing_post_BAO_H070p6_JLA` best fits from the *Planck 2015* release. We have taken into account 10^4 cosmological models, derived from varying the cosmological parameters consistently within the 1σ confidence intervals defined for the previous parametrization. The CPL parametrization is only one of the many dark energy phenomenological models available, but it is somewhat used as a reference model in the literature. Moreover, the large errors on its parameters, in particular on the dynamical dark energy EoS parameter w_a, make us confident on having explored a very large set of cosmological scenarios compatible with observational data, thus making our estimation for the range of z_M highly conservative. For this reason, we also consider a much more restrictive cosmological constant case (baseline model) which is recognized, at the preset stage of observations, as the best consensus cosmological model. At the end, it results that for the CPL case, z_M lies in the range $[1.4, 1.75]$ for more than 99% of 10^4 random cosmological models chosen as described above, while for the ΛCDM case, z_M lies in the range $[1.57, 1.62]$.

Then, after having performed steps from 1 to 6, we end up with the value of z_M which will be very likely measured by the future surveys we have chosen to work with, namely, *Euclid* and SKA. And once we have found the value of z_M, we can proceed to calculate the value of $c(z_M)$, if independent D_A and H are provided. That is actually possible, even within the same galaxy survey: while D_A can be provided by BAO, independent H measurements can be provided by a class of galaxies, mainly early type galaxies (ETG), which can serve as cosmic chronometers [32, 33]. Combining them, we can finally define a dimensionless parameter Δ_c as

$$\Delta_c(z_M) \equiv \frac{c(z_M)}{c_0} = D_A(z_M)\frac{H(z_M)}{c_0} \,. \tag{10}$$

Such parameter, as we have stated above and as can be derived from Eq. (5), is equal to 1 only if $c(z)$ is constant and equal to c_0; otherwise, any statistically well-based deviation from such value is an indication of a varying speed of light.

Final results are summarized in Table 1. In the first column, we define the VSL signal we have considered. In the second column, we report the maximum redshift corresponding to such models; note the variation in its error, which depends on the accuracy of the survey. In the third column we basically count how many simulations have a $c \neq c_0$ at 1σ confidence level; we derive its median value, the corresponding error (from the entire ensemble of $\mathcal{N}$ simulations), and we report the probability to detect a VSL at 1σ confidence level (numbers in parenthesis). The fourth and fifth columns are the same as the third one, but for the 2σ and 3σ confidence levels.

It is possible to check that SKA will be able to unequivocally detect a 1% variation in the speed of light, if any, at 3σ level (the reported probability is equal to 1); *Euclid* won't be able to detect the same signal, with only a $\sim 0.3\%$ probability of detection at 1σ; and smaller signals will be hardly detected at all, given the accuracy of the present planned surveys. This does not exclude that, in the future, we might be able to improve such accuracy at levels good enough to detect smaller VSL signals.

Table 1: Results from the maximum redshift method, Ref. [**SA5**].

		Euclid		
$\Delta c/c_0$	z_M	$c_{1\sigma}$ $(p_{>1})$	$c_{2\sigma}$ $(p_{>1})$	$c_{3\sigma}$ $(p_{>1})$
1%	$1.559^{+0.054}_{-0.051}$	$0.99993^{+0.00013}_{-0.00024}$ (0.32)	$0.99436^{+0.00023}_{-0.00041}$ (0)	$0.98879^{+0.00032}_{-0.00056}$ (0)
0.1%	$1.587^{+0.058}_{-0.052}$	$0.99199^{+0.00014}_{-0.00024}$ (0.001)	$0.98636^{+0.00024}_{-0.00038}$ (0)	$0.98072^{+0.00034}_{-0.00053}$ (0)
		SKA		
$\Delta c/c_0$	z_M	$c_{1\sigma}$ $(p_{>1})$	$c_{2\sigma}$ $(p_{>1})$	$c_{3\sigma}$ $(p_{>1})$
1%	$1.561^{+0.017}_{-0.017}$	$1.00585^{+0.00003}_{-0.00003}$ (1)	$1.004036^{+0.00005}_{-0.00005}$ (1)	$1.00221^{+0.00008}_{-0.00009}$ (1)
0.1%	$1.590^{+0.018}_{-0.017}$	$0.99797^{+0.00003}_{-0.00003}$ (0)	$0.99612^{+0.00006}_{-0.00006}$ (0)	$0.99428^{+0.00008}_{-0.00008}$ (0)

While the method we have described so far is an absolute novelty in the panorama of observational probes of alternative theories and it is the most promising attempt, so far, for a clear detection of a possible VSL signal at cosmological levels, it is also true that it has two main limitations in the form we have described above:

- the measurement of the speed of light is performed in one single point, corresponding to the redshift where the angular diameter distance reaches its maximum;
- the core equations for the method, i.e. Eqs. (5) and (10) are derived from the definition of the angular diameter distance, Eq. (4), which is based on the assumption of spatial flatness. While this sounds as a very likely possibility, giving our present observational knowledge [23], a generalization to any possible spatial geometry would be advisable.

In Ref. [**SA2**] we have gone exactly in those directions. First of all, we have found out a way to generalize the method and to measure the speed of light at any redshift possible from the range made available by the appropriate observations. The solution, again, comes from the same definition of the angular diameter distance and its relation to the expansion rate.

To start, we need the observational data which are available from future BAO galaxy surveys: the angular diameter distance (D_A) and the expansion rate (H). We define as D_A^{real} and H^{real} the results of such observations, i.e. the *numbers* that outcome the measurement process. This means that, starting from the theoretical definition of the angular diameter distance, Eq. (4), we assume that

$$D_A(z) \equiv D_A^{real} \doteq \frac{1}{1+z} \int_0^z \frac{c(z')}{H^{real}(z')} dz', \tag{11}$$

i.e., that the theoretical D_A function, ignoring what is on the right hand side of Eq. (11)), is *explicitly* equal to the function that can be directly obtained by observations. On the other hand, we can also assume that the *unknown* theoretical $H(z)$ is *explicitly* equal to the function that can be obtained by observations, H^{real}. Actually, these are much more than just assumptions: observations always bring signatures of the real underlying cosmological model, whose ignorance we parameterize in many ways. A very important point of this approach is that we do not need any cosmological assumption for $H(z)$, because we will directly use the output from the observations, $D_A^{real}(z)$ and $H^{real}(z)$, in order to calculate all the quantities we will define. This also means that we are loosing any possibility to recover any information on the cosmological model; but, if we change our perspective and strictly look at VSL theories, then the method will soon reveal its benefits.

The main point here is that we don't know, *a priori*, if the speed of light appearing in Eq. (11) is constant or not. In fact the question is: what if we have a *real* VSL signal to be detected? We can discover it by building two quantities. First, we calculate the derivative of the real observed D_A^{real}:

$$y_r^{real}(z) \doteq \frac{\partial}{\partial z}\left[(1+z) D_A^{real}(z)\right] \equiv \frac{c(z)}{H^{real}(z)} \ , \tag{12}$$

where, again, we have identified the unknown theoretical $H(z)$ function in Eq. (11) with the observed $H^{real}(z)$. Note that the quantity we calculate and use is y_r^{real}, which we know should be equal to $c(z)/H^{real}$ but, if we don't have an independent H measurement, we are unable to discriminate between changes in c or H^{real}. Then, we also have a *reconstructed* set of

$$y_r^{rec}(z) \doteq \frac{c_0}{H^{real}(z)} \ , \tag{13}$$

where we use independent measurements of H and make an explicit assumption for a constant speed of light in order to convert time observations (H) into distances (y_r). Thus, if we find that

$$y_r^{real}(z) = y_r^{rec}(z) \ , \tag{14}$$

it will mean that then the assumption we have made to built data from Eq. (13), i.e. that the speed of light is constant, is well based. On the contrary, if

$$y_r^{real}(z) \neq y_r^{rec}(z) \ , \tag{15}$$

then $c(z) \neq c_0$. What is important to stress is that in this case we can directly obtain (or reconstruct) the redshift function $c(z)$, through the ratio:

$$\frac{y_r^{real}}{y_r^{rec}} = \frac{c(z)}{c_0} = \Delta_c(z) \ . \tag{16}$$

As we have already pointed out above, in this way we are also circumventing the "dimensionless vs dimensional measurement" debate, because we are going to reconstruct a (dimensionless) relative variation of the speed of light, not an absolute (dimensional) quantity. It is also straightforward to check that Eq. (16) is nothing more than a generalization of the previously defined Eq. (10), which is now evaluated at any redshift we want, and not only at the maximum one.

Thus, using exactly the same algorithm we have described above, namely using steps 1-2-3 and 5, we can reconstruct a redshift extended VSL signal from galaxy surveys, using mock data from future surveys. Actually, the only difference is that in step 2 we have not used Gaussian Processes to reconstruct D_A and H: y_r^{real} is calculated as the derivative with respect to redshift of the observed quantity (D_A^{real}), which is represented by a discrete set of points (observations) which have an intrinsic dispersion around the underlying fiducial cosmological model. The problems related to the dispersion cannot be avoided: the dispersion is intrinsic to the measurement process, and we can only hope to have, in the future, better measurements which can reduce it. But we will always have an intrinsic systematic error in the derivation of y_r^{real}. Moreover, the dispersion alters the derivative calculation and thus, as it is known and expected, the errors on the derivated quantity tend to explode. Having assumed that this problem cannot be avoided, we can rely on another property of our approach: given that we are not interested in the explicit form of H, because we will directly use observations to infer a function which *interpolates* them, we are not forced to fit our quantities following some cosmological-model-based requirements. Thus, we can try a fit based on the best analytic functions which can work in this situation. In our case, we need analytic functions for fitting both H^{real} and D_A^{real}, and they have different requirements. For H^{real} we have found that a simple sixth-order redshift polynomial gives an optimal fit to H^{real} in the redshift range we will cover, i.e. $z \in [0.05, 2.75]$; higher-order polynomials do not improve the fit. As only general prior, we ask that $H(z) > 0$ all over the redshift range $z \in [0, \infty)$. For D_A^{real} a polynomial fit is unsatisfactory to describe the peculiar property of the angular diameter distance to have a maximum at relatively low redshift values; abetter and more flexible fit is given by the Padé approximant:

$$D_A^{real}(z) = \frac{d_1^t\, z}{1 + d_1^b\, z + d_2^b\, z^2}\ , \tag{17}$$

which clearly satisfies the expected condition $D_A^{real} = 0$ for $z \to 0$ and $z \to \infty$; moreover, we require that $D_A^{real} > 0$ for $z \in [0, \infty)$. Once the fits are run for both H^{real} and D_A^{real}, we have a set of parameters (the parameters of the polynomial and of the Padé approximant), respectively, with their covariance matrix and errors bars; after the correct propagation error rules are applied, we end up with a set of polynomial-reconstructed y_r^{real} and y_r^{rec}, with related errors, from which we can derive the $c(z)/c_0$ ratio through Eq. (16). Finally, this last quantity can also be fitted (or reconstructed); the function we have been working with is the Padé approximant given by:

$$\frac{c(z)}{c_0} = \frac{1 + c_1^t\, z}{1 + c_1^b\, z + c_2^b\, z^2}\ , \tag{18}$$

imposing the conditions: $c(z = 0)/c_0 = 1$, and that $c(z)$ is always positive for $z \in [0, \infty)$. We have verified that the functions we have finally chosen to fit H^{real} and D_A^{real} are really good approximations to the fiducial model all over the entire redshift range $z \in [0, \infty)$, and not only in the redshift interval we have decided to work with because covered by next galaxy surveys. On the other hand, the function chosen for $c(z)$ has some degree of arbitrariness: it describes very well our input VSL in the galaxy surveys redshift range, but not at very high redshifts. But it is very general, and with such a high level of flexibility that it can be used as a testing function to detect if a VSL signal is working or not in any case.

Moreover, we have focused on more surveys than before, namely: BOSS, DESI[7], *WFIRST-2.4*[8] and SKA, because, in their respective redshift ranges, they show the best performances. For BOSS, we have considered $z = 0.05$; for DESI, $z \in [0.15, 0.55]$; for *WFIRST-2.4*, $z \in [1.95, 2.75]$ and for SKA $z \in [0.65, 1.85]$.

[7]http://desi.lbl.gov/.

[8]http://wfirst.gsfc.nasa.gov/.

Results of such analysis are shown in Fig. 1, where we show visually (for the $\Delta c/c_0 = 1\%$ case) what we have previously described in Table 1. In red we show how many simulations, among the $\mathcal{N}$ we have produced, are able to detect a VSL signal at 1σ confidence level; in green and yellow we show detections at 2 and 3σ, respectively. It can be easily checked that in the redshift range $[0.85, 1.45]$, which is approximately covered by SKA, we are able to detect a 1% VSL signal at 3σ confidence level in 100% of our simulations.

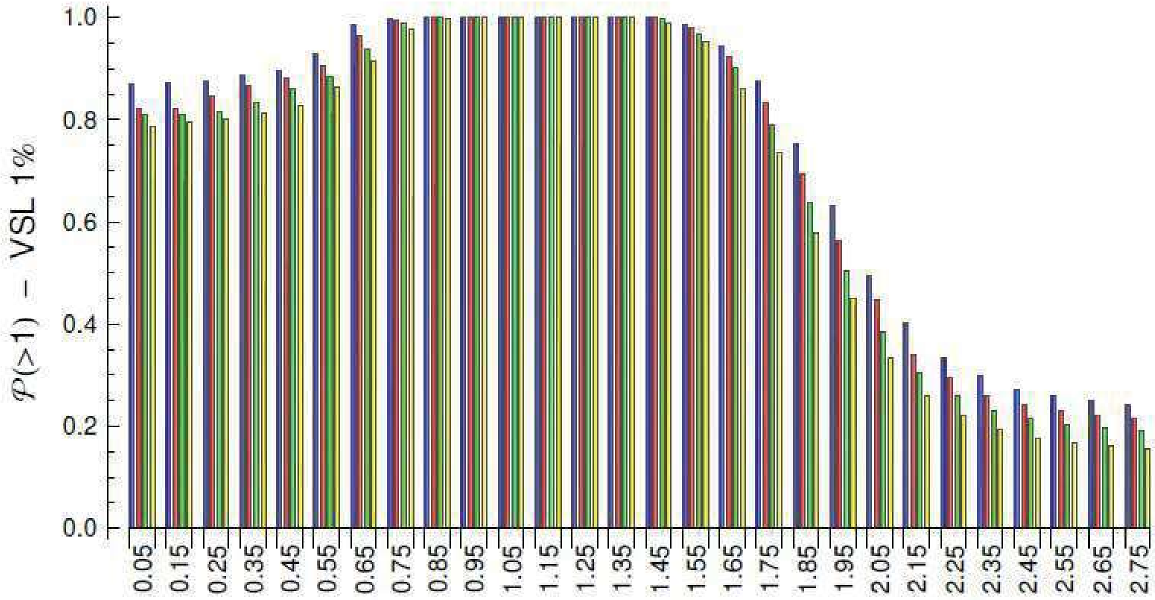

Figure 1: Probability to detect positive residuals of post-fitting reconstructed $c(z)$ vs. $c(z) = c_0$. Blue: residuals calculated from the best fit reconstructed values; red: residuals calculated from the 1σ lower confidence level from the reconstructed values; green: residuals calculated from the 2σ lower confidence level from the reconstructed values; yellow: residuals calculated from the 3σ lower confidence level from the reconstructed values.

Now, we can also generalize our results to the case where the universe is not spatially flat. If the curvature can vary, then the definition of the angular diameter distance is

$$D_A(z) = \begin{cases} \frac{D_H}{\sqrt{\Omega_k}(1+z)} \sinh\left(\frac{\sqrt{\Omega_k} D_C(z)}{D_H}\right) & \text{for } \Omega_k > 0 \\ \frac{D_C(z)}{1+z} & \text{for } \Omega_k = 0 \\ \frac{D_H}{\sqrt{|\Omega_k|}(1+z)} \sin\left(\frac{\sqrt{|\Omega_k|} D_C(z)}{D_H}\right) & \text{for } \Omega_k < 0\,, \end{cases} \tag{19}$$

where $\Omega_k \equiv k c_0^2/H_0^2$ is the dimensionless curvature density parameter today; $D_H = c_0/H_0$ is the Hubble distance; and the radial comoving distance is defined as $D_C(z) = D_H \int_0^z \mathcal{F}_c(z')/E(z') dz'$, where we have made use of the general ansatz $c(z) \equiv c_0 \mathcal{F}_c(z)$, with $\mathcal{F}_c(z) = 1$ for $z = 0$. We are assuming here the most general case of a varying speed of light $c(z)$; but the standard scenario can be easily recovered simply replacing $c(z)$ with c_0 any time it appears. If we now calculate y_r^{real} through the same Eq. (12), we have:

$$y_r^{real}(z) \equiv \begin{cases} \frac{c(z)}{H(z)} \cosh\left(\frac{\sqrt{\Omega_k} D_C(z)}{D_H}\right) & \text{for } \Omega_k > 0 \\ \frac{c(z)}{H(z)} & \text{for } \Omega_k = 0 \\ \frac{c(z)}{H(z)} \cos\left(\frac{\sqrt{|\Omega_k|} D_C(z)}{D_H}\right) & \text{for } \Omega_k < 0\,. \end{cases} \tag{20}$$

The most important point to be noted is that even if we assume $c(z) = c_0$, we would still have some contribution from the $\Omega_k \neq 0$ term; thus the case "VSL + spatial flatness" would be equivalent to "constant $c(z)$ + curvature". We can easily quantify how much information we might derive and erroneously attribute to a VSL signal only, and which should instead be shared with a non-null curvature signal. In particular, the previous Eq. (10) will be generalized to

$$\frac{D_A(z)H(z)}{c_0} = \Delta_c(z) \cdot \Delta_k(z) \;, \tag{21}$$

where

$$\Delta_k(z) = \begin{cases} \cosh\left(\sqrt{\Omega_k}\frac{D_C(z)}{D_H}\right) & \text{for } \Omega_k > 0 \\ 1 & \text{for } \Omega_k = 0 \\ \cos\left(\sqrt{|\Omega_k|}\frac{D_C(z)}{D_H}\right) & \text{for } \Omega_k < 0\,. \end{cases} \tag{22}$$

As before, Δ_c is, basically, the contribution to the signal given only by the relative variation of the speed of light between now and the redshift z epoch, while Δ_k quantifies the contribution to the signal from the spatial curvature. Actually, in Δ_k there is still some influence from $c(z)$, through the comoving distance D_C.

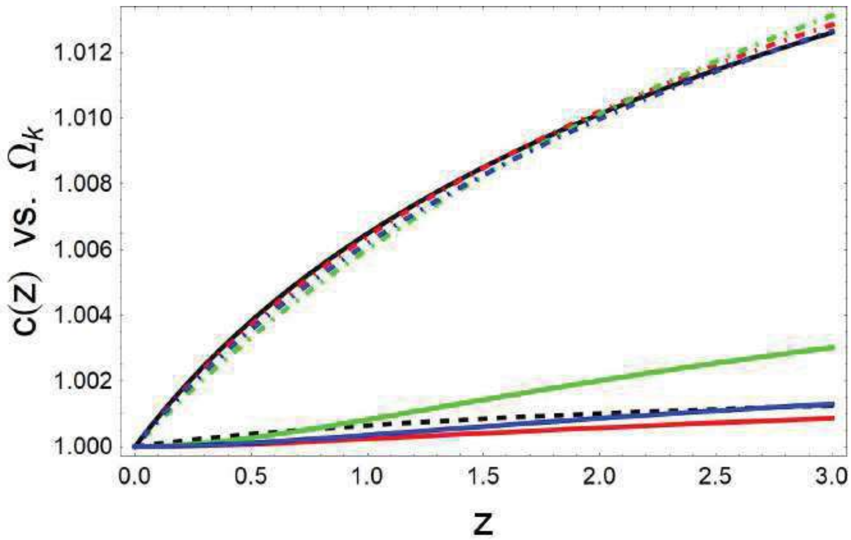

Figure 2: VSL vs. spatial curvature degeneracy displayed using Eqs. (21) - (22). Black lines: solid - 1% VSL signal plus null curvature; dashed - 0.1% VSL plus null curvature. Red lines: solid - correction from curvature term in Eq. (22) when $\Omega_k = 0.0008$ and assuming $c(z) = c_0$; dot-dashed - correction from curvature term in Eq. (22) when $\Omega_k = 0.0008$ and assuming a 0.95% VSL signal. Blue lines: solid - correction from curvature term in Eq. (22) when $\Omega_k = -0.0012$ and assuming $c(z) = c_0$; dot-dashed - correction from curvature term in Eq. (22) when $\Omega_k = 0.0008$ and assuming a 0.95% VSL signal. Green lines: solid - correction from curvature term in Eq. (22) when $\Omega_k = 0.0028$ and assuming $c(z) = c_0$; dot-dashed - correction from curvature term in Eq. (22) when $\Omega_k = 0.0008$ and assuming a 0.85% VSL signal.

The latest constraints from *Planck* on the spatial curvature is $\Omega_k = 0.0008 \pm 0.002$ at the 68% confidence level (and ± 0.004 at the 95%); we can thus compare the contribution from Δ_c and Δ_k. Results are shown in Fig. 2. In order to be as much complete as possible, we have also analyzed the lower and upper limits for the curvature parameters, i.e. $\Omega_k = -0.0012$ and $\Omega_k = 0.0028$.

Two main conclusions can be derived from it:

- a realistic contribution from the spatial curvature to our method would be $\lesssim 0.06\%$ at the maximum in D_A (for a more direct and straightforward comparison, we use the same maximum criterium we have used to define the 1% and the 0.1% VSL models) for both $\Omega_k = 0.0008$ (solid red line) and $\Omega_k = -0.0012$ (solid blue line). This would be even smaller than the 0.1% VSL signal (black dashed line) we have found out to be finally undetectable. The upper limit $\Omega_k = 0.0028$, would give a $\sim 0.15\%$ contribution; a slightly larger value, but still out of any possible detection with SKA;

- in general, a pure VSL and a pure curvature signal are degenerate. We can detect a total signal, without being able to ascribe it to one or another. What we can assess is that, given present bounds on curvature, a 1% signal (solid black line) could be attributed with no doubt to VSL only, rather than to any curvature contribution. Assuming both VSL and non-null curvature, given the actual constraints on the latter one, the VSL signal might be $\sim 0.95\%$ for $\Omega_k = 0.0008$ (dot-dashed red line) and $\Omega_k = -0.0012$ (dot-dashed blue line), and $\sim 0.85\%$ for $\Omega_k = 0.0028$ (dot-dashed blue line), in order to have a final total 1% detection. Thus, at least at the scales which we have shown to be directly testable in the next future, curvature might play a negligible role. But if the total signal should result to be less than 1%, then we could have problems and would not be able to discriminate between them.

Finally, we want to discuss another possible use of Eq. (5). When trying to assess if a cosmological model is really feasible, the first step is always to compare it with data. If the fit is good, one can generally assert that the model works. But is this enough? Of course not. That's why there is the need for some statistical tools which can be used to establish if a model is really statistically favoured with respect to another or not, like the Information Criteria (the Akaike Information Criterion [34, 35], the Residual Information Criterion [36], the Bayesian Information Criterion [37], the Deviance Information Criterion [38]) or the Bayesian Evidence [39, 40]. But even in this case, it might not be enough: after all, statistics needs some rules to work, and such rules are unable to really discriminate between what is real and what is not. For example, it is well known that the most accepted solution to explain the present acceleration at cosmological scales is the cosmological constant. It is very hard to beat it, on a statistical ground, because any model which is proposed as an alternative, for how simple it could be, will be always more complex than a constant, and will be statistically disfavoured. Thus, after a good fit, and a statistical comparison, we need a smoking-gun: a strikingly clear and net observational difference, which can help to tip the balance in favour of a model or another. In Ref. [**SA1**] we have found out that Eq. (5) might also be used as such smoking-gun.

In Ref. [**SA1**] we try to go over the most standard assumptions which underlie most of the models which are proposed today. In particular, most of them are based on the assumption of the isotropy and homogeneity of the universe on large scales; while this is tested to be exact down to some very large scale, it is obvious that, at some point, inhomogeneities appear. And there might be much more clues pointing toward it, as long as some inhomogeneities are observationally detected, for example: the $\alpha-$dipole, i.e. spatial variations in the fine structure constant from quasars [41, 42, 43, 44, 45, 46]; the dark flow dipole, detected by [47] using peculiar velocity measurements from galaxies, and by [48, 49, 50] using the imprinting of clusters of galaxies velocity in the cosmic microwave background through Sunyaev-Zeldovich signal; the dark energy dipole found by [51] using both quasars and type Ia supernovae. This may suggest some large-scale inhomogeneous distribution of matter in the universe which could perhaps be explained by allowing an inhomogeneous model of the universe. All these detections are, anyway, puzzling and debated: improvements in the distance estimators can mitigate the dark flow dipole from galaxy velocities, but still being significant at a 98% level [52]; officially, the *Planck* team does not find any statistical evidence for dark flow [53], but [54, 55] still claim on it; while for what concerns the dark energy dipole, the statistical usefulness of supernovae has been found out to be null by [56, 57].

In more detail, in Ref. [**SA1**] we have studied the possibility that cosmological inhomogeneities might be related to the inhomogeneous pressure model of Stephani [58]. This general model has already been investigated, both theoretically [59, 60] and observationally [61, 62, 63, 64, 65, 66], imposing relatively strict bounds on the inhomogeneity, though not eliminating it completely. We explore the possibility that such inhomogeneity might be perhaps the reason for the $\alpha-$dipole but through a $c-$dipole. The Stephani universe is an inhomogeneous perfect-fluid energy-momentum tensor conformally flat solution of the Einstein field equations with a general spherically symmetric metric given by [58, 59]

$$ds^2 = -c_0^2 \frac{a^2}{\dot{a}^2} \left[\frac{\left(\frac{V}{a}\right)^{\cdot}}{\left(\frac{V}{a}\right)} \right]^2 dt^2 + \frac{a^2}{V^2} \left[dr^2 + r^2 d\Omega^2 \right], \tag{23}$$

where

$$V(t,r) = 1 + \frac{1}{4} k(t) r^2 , \tag{24}$$

and $(\ldots)^{\cdot} \equiv \partial/\partial t$. The function $a(t)$ plays the role of a generalized scale factor, $k(t)$ has the meaning of a time-dependent "curvature index", r is the radial comoving coordinate, and c_0 is the (constant) speed of light. The energy density and pressure are given by

$$\varrho(t) = \frac{3}{8\pi G} \left[\frac{\dot{a}^2(t)}{a^2(t)} + \frac{k(t) c_0^2}{a^2(t)} \right], \tag{25}$$

$$p(t,r) = w_{eff}(t,r) \varrho(t) c_0^2 = \left[-1 + \frac{1}{3} \frac{\dot{\varrho}(t)}{\varrho(t)} \frac{\left[\frac{V(t,r)}{a(t)}\right]}{\left[\frac{V(t,r)}{a(t)}\right]^{\cdot}} \right] \varrho(t) c_0^2, \tag{26}$$

and generalize the standard Einstein-Friedmann equations into inhomogeneous models. We will focus on the model of Eq. (23), following [59, 60, 66], with $k(t) = \beta a(t)$ and $\beta = const.$, which simplifies the metric to

$$ds^2 = -\frac{c_0^2}{V^2} dt^2 + \frac{a^2(t)}{V^2} \left(dr^2 + r^2 d\Omega^2 \right). \tag{27}$$

The point is that this metric can be considered as defining spatially dependent effective speed of light $c(t,r) = c_0/V(t,r)$ or still can mimic the spatial dependence of the speed of light provided we take $c_0 \to c = c(t)$ in Eq. (27) and make an appropriate ansatz. Our choice is in some way strategic, because we have selected three different expressions for a VSL which stand for three different ways for VSL and inhomogeneity to be entangled:

- standard (classical) no-varying c: $c(t) = c_0 =$ const. In this case, we have no VSL, but only inhomogeneity;
- Barrow-Magueijo ansatz, from [13]: $c(t) = c_0 a^n(t)$, where we assume separate temporal behaviors for both VSL and inhomogeneity (which is also spatially dependent);
- "inhomogeneous" ansatz: $c(t,r) = c_0/V(r,t)$, where we assume that the time variation of VSL is intrinsically correlated to the inhomogeneity.

Given that the redshift in the Stephani model is defined as

$$1 + z = \frac{a_0}{a_e} \frac{V_e}{V_0} , \tag{28}$$

the angular diameter distance reads as

$$D_A = \frac{a(t)}{V(t,r)} r = \frac{a_0}{V_0(1+z)} r \ , \tag{29}$$

with the radial comoving distance being, as usual

$$r = \int_{t_e}^{t_0} \frac{c(t)dt}{a(t)}. \tag{30}$$

We can now derive the condition for the maximum of the angular diameter distance, ending up with a modified version of Eq. (5):

$$c(a,r) = \frac{D_A(a)H(a)}{1 + \frac{\Omega_{\beta,0}}{2}\, a\, r^2(a)} \ , \tag{31}$$

or, equivalently, we can redefine/generalize Eq. (10) into:

$$\Delta_c(a,r) = \frac{D_A(a)H(a)}{c_0} = \begin{cases} 1 + \frac{\Omega_{\beta,0}}{2}\, a\, r^2(a) & \text{for } c(t) = const. \\ a^n \left(1 + \frac{\Omega_{\beta,0}}{2}\, a\, r^2(a)\right) & \text{for } c(t) = c_0 a^n(t) \\ \left[\frac{1+\frac{\Omega_{\beta,0}}{2}\, a\, r^2(a)}{1-\frac{\Omega_{\beta,0}}{4}\, a\, r^2(a)}\right] & \text{for } c(t,r) = \frac{c_0}{V(r,t)} \end{cases} \tag{32}$$

We can note that even in the first case, with constant speed of light, the inhomogeneity might play the role of an "effective" VSL; in particular, inhomogeneity might mimic a time and space varying speed of light. For the other two ansätze, it would be actually impossible to discriminate between a pure VSL signal and a pure inhomogeneity, because the two are strongly coupled.

Anyway, the main point is another one: we find that the Stephani model is able to match observational data as good as ΛCDM, but it is slightly disfavoured by statistical tools, because it has more parameters. Then, it is useful to ask: is there any peculiar signature which makes the Stephani different and distinguishable from the standard scenario in a clear way? The solution is exactly in the maximum redshift value, and the parameter Δ_c defined in Eq. (32).

Table 2: Results of the cosmological fit for the Stephani model, Ref. [**SA1**].

	H_0	Ω_β	w	n	z_M	Δ_c
$c(t) = c_0 = const.$	$69.6^{+0.7}_{-0.7}$	$0.682^{+0.022}_{-0.023}$	$-0.014^{+0.004}_{-0.004}$	–	1.553 ± 0.026	1.140 ± 0.011
$c(t) = c_0 a^n(t)$	$69.6^{+0.7}_{-0.6}$	$0.638^{+0.031}_{-0.029}$	$-0.139^{+0.047}_{-0.045}$	$-0.083^{+0.034}_{-0.034}$	1.816 ± 0.132	1.281 ± 0.074
$c(t,r) = c_0/V(r,t)$	$69.6^{+0.7}_{-0.7}$	$0.669^{+0.022}_{-0.022}$	$0.003^{+0.003}_{-0.003}$	–	1.708 ± 0.042	1.200 ± 0.015

In Table 2 we report the values for all the theoretical parameters, and for the derived ones, i.e. z_M and $\Delta_c(z_M)$, obtained by fitting each one of the previous VSL ansätze in a Stephani universe with different cosmological probes. We have used Type Ia Supernovae [67]; BAO data from [68]; and *Planck* 2015 Cosmic Microwave Background data from [69]. We can see that while the cosmological parameters for each model are clearly statistically consistent with each other, the maximum redshift is located at very different points, and the deviation from the standard value $\Delta_c = 1$ is absolutely measurable. Given the bounds from present data, for the ΛCDM case, z_M lies in the range $[1.57, 1.62]$ (3σ confidence level); thus, when comparing this range with our results in Table 2 we infer that in the first case, the model would be ruled out at almost 3σ; while the second and third ansätze would be completely discarded. Moreover, as we have discussed above and in Refs. [**SA5**,**SA6**], in principle, SKA will be able to detect a 1% deviation from constant speed of light at 3σ confidence level at the maximum redshift. It is clear that all the models we have considered here exhibit variations which are fully detectable, being of the order of the 10%. Thus, the models we have considered have the good quality of being completely falsifiable: if no signal of such order of magnitude will be detected, it will be a clear signature that no cosmological inhomogeneity is at play. Still, a VSL might be possible, but not a spatial inhomogeneity.

4.1.4 Modifying gravity in clusters of galaxies

While the variation of fundamental constants is a collateral effect of the attempt to generalize GR, mainly due to the introduction of new fields with possible peculiar properties, the most straightforward and common way to test departures from GR is by analyzing all the phenomena which are more directly related to gravitational effects. From this point of view, the latest and most precise sets of measurements concerning the dynamics of our Universe are those from the second release of the *Planck* satellite [23, 71], which have confirmed that the ΛCDM model has to be considered as the best model to explain most of the phenomena occurring in it. Nevertheless, it is undisputable that it also has many problems [9]. For what we are interested in, the ΛCDM paradigm is based on: the cosmological constant (CC), introduced to explain the accelerated expansion of our Universe detected for the first time by means of Type Ia Supernovae in [72]; the dark matter, as the main ingredient of large scale structure formation and evolution; and on the acceptance of GR as the theory of gravity. The intrinsic simplicity of the CC makes it difficult to be confirmed or refuted on a purely statistical base, even if we basically ignore what it could be its origin; and we still lack a direct laboratory detection of one of the many suitable candidates for DM. Finally, GR endures any challenge and has passed any test it is undergone [73, 74]. But both the DM and the CC problems (as well as the generalization of the CC to a time-dependent fluid, the dark energy) might be closely connected due to the adoption of GR. Thus, overcoming GR might help to solve them. Unfortunately, extensions or modifications of GR can be performed in too many ways [75]; but GR is a very well-tested theory at Solar System scales [76], and this poses very strong and limiting bounds on any possible generalization. It is also well-known that most of these theories are predominantly introduced to consistently extend the CC scenario and give a well-based theoretical background for dark energy's nature, i.e., to give an explanation for present acceleration of the universe on cosmological scales. And they turn out to be, at the present stage of cosmological observations, basically indistinguishable from the same problem they would like to solve.

Among the plethora of models that have been proposed so far, theories which exhibit a *screening mechanism* are gaining much interest lately. Basically, most of the scenarios which are proposed require, at least, a scalar field coupled to matter and mediating a so-called "fifth-force" which, in principle, might span the entire range from Solar System up to cosmological scales. In regions of high density, this force has to be self-suppressed, so that no deviation from GR should be operative or, at least, if there was any, it should be hardly detectable, with laboratory experiments able to establish, generally, only an upper limit of detection for it [77]. Instead, where the density is lower, the modification to GR should start to be effective and possibly some observational signatures might arise. What kind of and what order might these signatures be, depends on the model.

There are two main leading ideas behind my interest in these theories exhibiting a screening mechanism:

- the first one: the scale at and the way in which such screening mechanisms can (could) vanish or break, should induce observational signatures which make such theories clearly testable and falsifiable. In principle, we should be able to differentiate them from GR;

- the second one: we might be able to use these same mechanisms to find a connection between the cosmological and the astrophysical scales of both "dark sectors", respectively dark energy and dark matter. Thus, it would be interesting to check if these theories might also mimic dark matter on astrophysical scales, being opportunely screened at even smaller scales, like the Solar System ones, where their effects should be negligible.

In Refs. [**SA3**,**SA4**], we have decided to focus on a family of theories which exhibit this kind of screening mechanism: the Galileon fields. Galileon fields are so defined because, by construction, they are invariant under the galilean shift symmetry; their peculiarity is that, although being higher-derivative field theories, they still have second order equations of motion [78, 79, 80, 81]. The screening mechanism behind Galileon fields is called Vainshtein screening [82], and it is due to changes from the kinetic contribution of the field to the Lagrangian, with first or second order derivatives becoming important in a certain range.

Within the large class of Galileon theories, we have studied a particular variation, proposed by [83], whose main attraction is that the Vainshtein screening mechanism can be broken at some astrophysical scale and, as such, it could have some influence on the internal dynamics of gravitational structures. This relatively new sub-class of the Galileon family is defined by the Lagrangian [83]

$$\frac{\mathcal{L}}{\sqrt{-g}} = M_{Pl}^2 \left[\frac{R}{2} - \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \frac{\mathcal{L}_4}{\Lambda^4} \right] , \tag{33}$$

where g is the determinant of the metric; R is the Ricci scalar; Λ is a mass dimension scale/constant; $\mathcal{L}_4$ is defined as

$$\mathcal{L}_4 \equiv -X \left[(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu} \right] - \left(\phi^\mu \phi^\nu \phi_{\mu\nu} \Box\phi - \phi^\mu \phi_{\mu\nu} \phi_\rho \phi^{\rho\nu} \right) , \tag{34}$$

where ϕ is the Galileon field; $\phi_{\mu_1 \dots \mu_n} \equiv \nabla_{\mu_1} \dots \nabla_{\mu_n} \phi$ and $X \equiv -1/2 \partial_\mu \phi \partial^\mu \phi$ is the standard kinetic term; and the reduced Planck mass appearing in the Lagrangian is defined as $M_{Pl} = (8\pi G)^{-1}$, where G is the bare gravitational constant and can differ from the usually measured one, G_N. Assuming a metric signature $(-,+,+,+)$, and the Newtonian Gauge, the perturbed Friedmann-Lemaître-Robertson-Walker metric can be written as

$$\mathrm{d}s^2 = -\left[1 + 2\frac{\Phi(r,t)}{c^2} \right] c^2 dt^2 + a^2(t) \left[1 - 2\frac{\Psi(r,t)}{c^2} \right] \delta_{ij} \mathrm{d}x^i \mathrm{d}x^j , \tag{35}$$

where c is the speed of light (we are working with a standard constant speed of light here); a is the cosmological scale factor; and Φ and Ψ are the gravitational and the metric potentials. After having defined the parameter

$$\Upsilon \equiv \left(\frac{\dot{\phi}_0}{\Lambda} \right)^4 , \tag{36}$$

the model can be fully characterized by the following equations:

$$\frac{\mathrm{d}\Phi(r)}{\mathrm{d}r} = \frac{G_N M(r)}{r^2} + \frac{\Upsilon}{4} G_N M''(r) , \tag{37}$$

$$\frac{\mathrm{d}\Psi(r)}{\mathrm{d}r} = \frac{G_N M(r)}{r^2} - \frac{5\Upsilon}{4} \frac{G_N M'(r)}{r} , \tag{38}$$

where $M(r)$ is the mass enclosed in a radius r. Thus, the effect of this screening breaking is clear: it introduces non-linear deviations from the standard GR expressions of the gravitational and metric potentials, through some terms depending on the local density. Note that the theoretical parameter Υ quantifies how much different is the new theory from GR, which is restored in the limit $\Upsilon \to 0$. In the most general case, we can have two different Υ parameters, one for each potential (we will assume Υ_1 for Φ, and Υ_2 for Ψ).

Even at some smaller-than-cosmological scales, the modified Galileon might have some influence on the dynamics of galaxies or of clusters of galaxies. Starting from this point, our idea has been: *what if such screening breaking, in an alternative modified gravity scenario, might be able to mimic effects of dark matter from standard general relativity?*

We have chosen to focus on clusters of galaxies because they are the best probes in this case for two main reasons: first, because they are the largest smaller-than-cosmological-scale structures whose dynamics is well studied; second, because they are the best tools to employ gravitational lensing as observational probe. This is utterly important, because gravitational lensing is the best way to estimate the real mass of a cluster, being independent of local internal astrophysical phenomena which might induce wrong mass density estimations, not due to gravity but to local perturbations.

Actually, we have faced the problem in two different ways: in Ref. [**SA3**], we have assumed that the Galileon field (with one single Υ parameter) works only at cosmological scale, as a substitute of the dark energy, and we have studied its influence on the internal dynamics of the clusters of galaxies; in Ref. [**SA4**], instead, we have assumed that the same Galileon field, with two Υ parameters, might also play the role of dark matter, namely, we have tried to fit observational data available for clusters of galaxies assuming that the only mass they are made of is of baryonic nature (gas and galaxies), and no dark matter is present, but its effects are consequences of the interaction between the Galileon field and the baryons.

The data we have at our disposal are taken from the sample of clusters observed by the Cluster Lensing and Supernova survey with Hubble *CLASH* [84]: we have a total of 20 clusters for which we have mass estimations obtained in two different ways:

- from X-ray observations [85]: the hot intra-cluster gas is heated up to temperatures of the order of 10^3 K and emits in the X-ray wavelength band. Assuming spherical symmetry, and that the gas is in hydrostatic equilibrium, its dynamics can be described by the collisionless Boltzmann equation

$$-\frac{\mathrm{d}\Phi(r)}{\mathrm{d}r} = \frac{kT_{gas}(r)}{\mu m_p r}\left[\frac{\mathrm{d}\ln\rho_{gas}(r)}{\mathrm{d}\ln r} + \frac{\mathrm{d}\ln T_{gas}(r)}{\mathrm{d}\ln r}\right] , \tag{39}$$

  from which, in GR, assuming the standard gravitational potential Φ, we can simply obtain:

$$M_{tot}(r) = M_{gas}(r) + M_{gal}(r) + M_{DM}(r) = -\frac{kT_{gas}(r)}{\mu m_p G_N} r\left[\frac{\mathrm{d}\ln\rho_{gas}(r)}{\mathrm{d}\ln r} + \frac{\mathrm{d}\ln T_{gas}(r)}{\mathrm{d}\ln r}\right] . \tag{40}$$

  From the right-hand-side, making direct use of observations (gas density and temperature profiles), one can obtain the total mass in the cluster, M_{tot}; of course, from the observed density ρ_{gas}, one can also derive the mass of the hot gas, M_{gas}. Thus, Eq. (40) is generally used to indirectly infer properties of the dark matter halo embedding the cluster, M_{DM}.

- from gravitational lensing events [86]: it is well-known that gravity can bend light, and from the patterns/images which are produced we are able to reconstruct the mass distribution of the cluster. The quantity which is generally reconstructed from the gravitational lensing events from a gravitational source is called convergence as is defined as

$$\kappa(R) = \frac{1}{c^2}\frac{D_l D_{ls}}{D_s}\int_{-\infty}^{+\infty} \nabla_r \left(\frac{\Phi(R,z)+\Psi(R,z)}{2}\right) dz\,, \tag{41}$$

where R is the two-dimensional projected radius; z is the line of sight direction; $r = \sqrt{R^2+z^2}$ is the three-dimensional radius; ∇_r is the Laplacian operator in spherical coordinates; and c is the speed of light. In GR, as known, $\Phi = \Psi$; but in general, they can be different, as it is the case of our Galileon model.

Then, what we have done in our analysis is:

- in Ref. [**SA3**] we have assumed that the gravitational potential on the left hand side of Eq. (39) is given by Eq. (37), and the potentials entering Eq. (41) are given by both Eq. (37) and (38). We have also considered one single common parameter Υ for both the potentials. Thus, we have tried to test if the Galileon modifications are compatible with the mass estimation M_{tot} derived from GR. Basically, we will obtain upper limits for how much we can modify gravity and still have a good match with observations. In order to perform this analysis, of course, we will deal again with all the classical component of a clusters, namely, galaxies, gas and dark matter, which will be described by the well-known and most commonly used Navarro-Frenk-White (NFW) profile [87]

$$\rho_{NFW} = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}\,, \tag{42}$$

  where the only free parameters are a density (ρ_s) and a scale (r_s);

- in Ref. [**SA4**], instead, we have assumed that the potentials given in Eqs. (37) - (38) have two different parameters, respectively Υ_1 and Υ_2; and that the only matter contribution to the cluster is given by the gas mass, which can be estimated from X-ray observations. With these ingredients, we have tried to fit the convergence profiles, which are mostly independent from the local internal dynamics of the clusters.

The main result of Ref. [**SA3**] is that Galileon theory is perfectly consistent with GR-based results. Given present observational accuracies, it is impossible to state a clear difference between the two approaches, which are statistically equivalent. But we have also found a possible dependence of the outcomes of our analysis with the dynamical internal status of a cluster, which would make the Galileon approach more viable than the classical GR to match observations. It is well known that a tension, in terms of mass estimations, between X-ray and lensing observations is present, basically, because X-ray estimations might be perturbed by local astrophysical phenomena which would affect the related mass measurements, while lensing is not affected at all by them, but only depends on the total gravitational potential. In our analysis, we have classified all the clusters from our sample in three groups: clusters for which the separate analysis from X-ray and lensing are consistent at 1σ level; those for which the tension is at least at 2σ level; and those with a tension higher than 3σ. It comes out that clusters which are more relaxed and, thus, whose X-ray profiles are less perturbed by possible astrophysical local processes, belong to first group; in this case, the galileon model gives a good fit to both X-ray and lensing observations and the parameter which quantifies the deviation from GR, Υ, is consistent with the GR value being $\lesssim 0.086$ at 1σ, $\lesssim 0.16$ at 2σ, and $\lesssim 0.23$ at 3σ. Anyway, statistically speaking, there is no clear evidence in favor of this model, with respect to GR; we can only assert that the Galileon model is as good as GR in order to explain observations, when considered as a cosmological scale fluid.

The interesting part is that custers from the other groups exhibit a much more positive and striking evidence in favor of the Galileon. In particular, it seems that the Galileon is more able to reduce the tension between X-ray and lensing data than GR, by mimicking in some way the physics behind it through the terms that are led by the Vainshtein breaking. But even in this case, in order to be statistically confident about such results, and to state in a more confident way that a real possible deviation from GR is operative, better data are needed. “Better” in this case means: to reduce some of the systematic uncertainties from calibration; to perform a better choice of modeling methods; to observe larger samples to limit scatter from relaxation state of the clusters or their asymmetry. It is also true that there could be astrophysical phenomena at nonlinear scales from baryonic physics that could be degenerate with Galileon effects, and such possible degeneracies with nonlinear baryonic effects should be studied and considered.

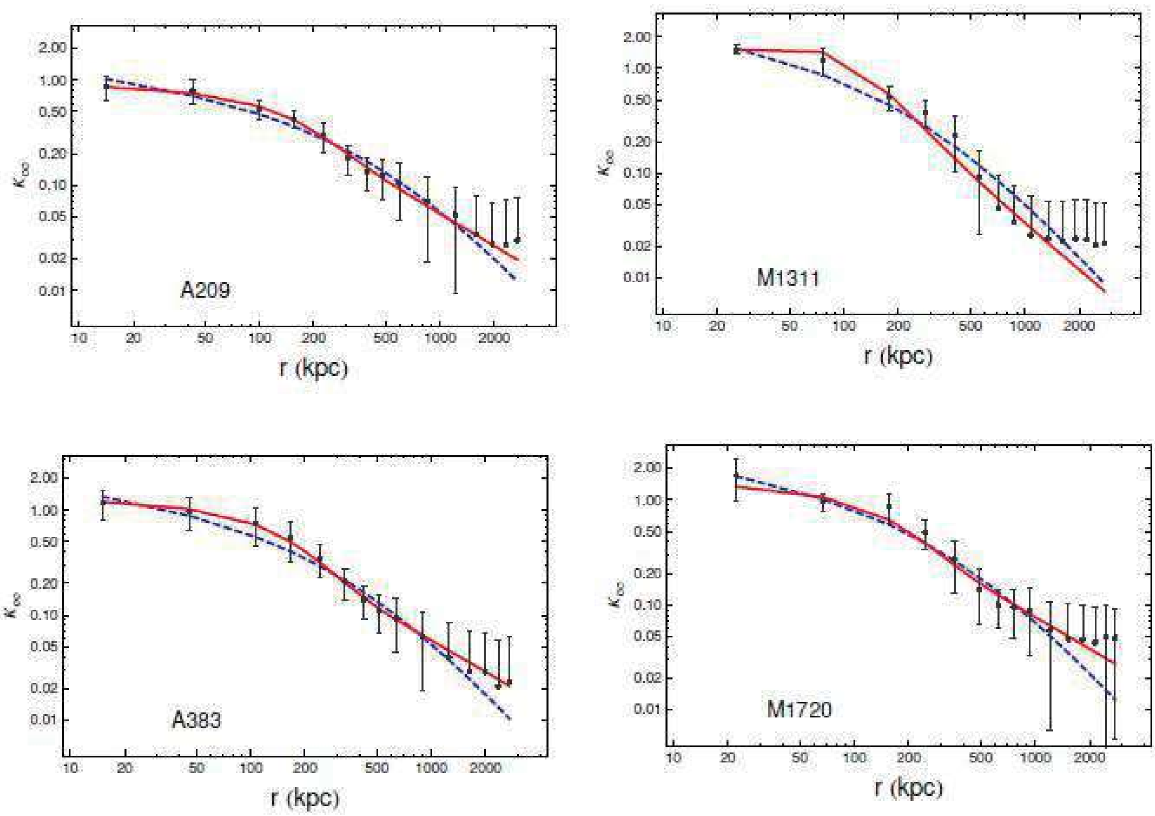

Figure 3: Convergence map from gravitational lensing reconstruction. Color code: grey points - observational data; dashed blue - GR + NFW; solid red - Galileon + gas.

Finally, in Ref. [**SA4**], we show that the Galileon model can be used to effectively mimic dark matter at clusters' scales. In particular, final results show that the Galileon model is even more statistically favourable than the GR to match lensing observations, when considered as a substitute of dark matter. From Fig. (3), where four representative cases are shown, one can see where the Galileon behaves better than GR: it fits better low-medium distances $\approx 100 - 200$ kpc, where it seems to better follow the decreasing trend of the convergence; and it also perform better at very large distances > 1 Mpc. Anyway, the latter have less statistical weight because of the larger errors; the main improvement comes from the low-medium distance range.

Thus, in principle, we have a theory (the Galileon), which could be used to play the role of dark energy at cosmological scales and, at the same time, the role of dark matter on astrophysical scales. But, is this really feasible? First, we need to point out that results from Ref. [**SA3**] cannot be connected "directly" with those from Ref. [**SA4**]:

- when the Galileon is compared with GR, the scale of the screening mechanism (or of its breaking) could be found out easily by checking where the correction terms (depending on Υ_1 and Υ_2) become important with respect to the "classical" Newtonian ones. But this makes sense - only - when comparing GR+dark matter+baryons and Galileon+dark matter+baryon, as in Ref. [**SA3**]. Instead, in Ref. [**SA4**], we have explored a completely different possibility, where the correction terms can play the role of dark matter over all the astrophysical scale range we have been considering. Actually, the Υ_1 and Υ_2-led terms are important at all scales, otherwise they would not be able to replace dark matter everywhere inside the cluster;

- the values of the parameter Υ are very different: in Ref. [**SA3**], the only Υ considered is $\mathcal{O}(0.1)$ and positive (by definition), while in Ref. [**SA4**] the two parameters are generally $\mathcal{O}(10)$, and can be of any sign. The former results are perfectly consistent with other constraints from literature obtained from stellar scales [88, 89]. The latter high values of the Υ parameters in Ref. [**SA4**], on the contrary, might lead to question if such values are really compatible with Solar System constraints, for which $\Upsilon \to 0$. But, are our results really not-consistent with Solar System constraints? We think the answer is no. If we had a star in the center of a galaxy, and another one in the outer arms, would we measure - locally - a violation of GR in any of them? We would say no, given that gravity is "self-similar"; we would always obtain the same constraints on Υ, independently of their location in the galaxy. Their gravitational well is in some way isolated - locally - on that scale. But such stars are embedded in a deeper (background) potential, that of the galaxy which leads their dynamics inside the same galaxy. And on that galactic scale we might have a break of GR, if we are ready to interpret dark matter not as a real matter component, but as a Galileon field, or a geometrical effect. That said, one better way to present our results could be: we have found that it is in principle possible to explain dark matter as a result of both Vainshtein and GR breaking, on scales as large as those tested (from 100 kpc to 2 Mpc). On smaller scales, in order to preserve GR, we need to fill the gap, i.e. we neeed to analyze smaller structures and find, lately, a gravitational structure which does not require Υ large, but ~ 0.1 or less, in order to be described (and this could easily happen, as we expect dark matter not to have any influence on Solar System scales). Finally, in this work we are just saying that the breaking scale might not be at cosmological scales (with Galileon playing the role of dark energy only), but at smaller ones too, albeit, much larger than Solar System;

- we also want to point out that in our analysis we have avoided (due to lack of measurements) the use of (at least) the central galaxies in the clusters, and this play an important role in describing the cluster potentials in the very inner regions. In fact, the addition of the galaxies has the potentiality to lower the values of the constants Υ_1 and Υ_2 (maybe to the safer $\mathcal{O}(1)$ order or lower) and, thus, to reduce the contribution from the Galileon in such regions.

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5 Scientific achievement not directly included in the habilitation procedure

5.1 Scientific publications in journals listed in Journal Citation Reports (JRC) database published after Ph.D. completion

SB1. Beltrán Jiménez J., Sáez-Gómez D., **Salzano V.**, Lazkoz R., *"Observational constraints on cosmological future singularities"*, Eur.Phys.J. C76 (2016) no.11, 631.

In this article we test some phenomenological models involving a future cosmological singularity, and we compar them to observational data to infer limits on the singularity time. My contribution consisted in: performing all the numerical calculations; discussing the results obtained; and giving an active contribution to the article writing.

My percentage contribution is estimated at about 35%.
Impact Factor: 4.912 (2016 JCR).

SB2. Beltrán Jiménez J., Rubiera-Garcia D., Sáez-Gómez D., **Salzano V.**, *"Cosmological future singularities in interacting dark energy models"*, Phys.Rev. D94 (2016) no.12, 123520.

In this article we discuss how most of the singularity scenarios in literature can be mapped into a singularity of an interaction between dark matter and dark energy and how, from this perspective, we can define a new kind of singularity not considered in the past. My contribution consisted in giving an active contribution to the article writing.

My percentage contribution is estimated at about 15%.
Impact Factor: 4.506 (2016 JCR).

SB3. Bull P., Akrami Y., ..., **Salzano V.** (♯31), *"Beyond ΛCDM: Problems, solutions, and the road ahead"*, Phys.Dark Univ. 12 (2016) 56-99.

This work is a sort of proceeding of the conference *"Beyond ΛCDM"* held in Oslo in January 2015, for which I was invited to co-chair one of the parallel discussion sessions, titled "Model selection vs. parameterizations: what do we expect to learn?". Results of the subsequent discussion have been reported in this document. My contribution consisted in giving an active contribution to the article writing.

My percentage contribution is estimated at about 2%.
Impact Factor: 5.222 (2016 JCR).

SB4. Lazkoz R., Leanizbarrutia I., **Salzano V.**, *"Cosmological constraints on fast transition unified dark energy and dark matter models"*, Phys.Rev. D93 (2016) no.4, 043537.

In this paper we explore the observational adequacy of a class of Unified Dark Energy/Matter models, where dark matter and dark energy are described by one single fluid exhibiting a fast phase transition from one form to another. We report how from the statistical point of view these models cannot be discarded when compared to the standard ΛCDM model. My contribution consisted in performing part of the numerical calculations; discussing the results obtained; and giving an active contribution to the article writing.

My percentage contribution is estimated at about 33%.
Impact Factor: 4.506 (2016 JCR).

SB5. Dąbrowski M.P., Gohar H., **Salzano V.**, *"Varying constants entropic-ΛCDM cosmology"*, Entropy 18(2), 60 (2016).

In this work we discuss the feasibility of a cosmological scenario where a cosmological entropic force is combined with the variation of some fundamental constants as the speed of light and the Newton gravitational constant. We set the theoretical background and we compare the model with observational data. My contribution consisted in performing all the numerical calculations; discussing the results obtained; and giving an active contribution to the article writing.

My percentage contribution is estimated at about 33%.
Impact Factor: 1.743 (2016 JCR).

SB6. Beltran Jimenez J., **Salzano V.**, Lazkoz R., *"Anisotropic expansion and SNIa: an open issue"*, Phys.Lett. B741 (2015) 168-177.

In this work we review the possibility of using Type Ia Supernovae observations to detect potential signatures of an anisotropic expansion in the Universe. We show that, contrary to the literature, when observational errors are taken in consideration in their entirety, this possibility is smoothed out, and the spatial distribution of the observations can bias this kind of analysis. My contribution consisted in performing all the numerical calculations; discussing the results obtained; and giving an active contribution to the article writing.

My percentage contribution is estimated at about 50%.
Impact Factor: 4.787 (2015 JCR).

SB7. Montiel A., **Salzano V.**, Lazkoz R., *"Observational constraints on the unified dark matter and dark energy model based on the quark bag model"*, Phys.Lett. B733 (2014) 209-216.

In this work we explore the compatibility with observations of the hypothesis that a small part of quarks and gluons did not yield to hadronization and resisted either as isolated aggregates of quark-gluon nuggets, behaving like dark matter, or as a perfect fluid in the form of a quark-gluon plasma, uniformly spread on cosmological scales and behaving as dark energy. My contribution consisted in performing part of the numerical calculations and giving an active contribution to the discussion of the results and to the article writing.

My percentage contribution is estimated at about 33%.
Impact Factor: 6.131 (2014 JCR).

SB8. Montiel A., Bretón N., **Salzano V.**, *"Parameter estimation of a nonlinear magnetic universe from observations"*, Gen.Rel.Grav. 46 (2014) 1758.

In this paper a cosmological model consisting of a nonlinear magnetic field coupled to a Robertson-Walker geometry is tested with observational data. My contribution consisted in supervising the numerical calculations and the writing of the results section.

My percentage contribution is estimated at about 15%.
Impact Factor: 1.771 (2014 JCR).

SB9. Montiel A., Lazkoz R., Sendra I., Escamilla-Rivera C., **Salzano V.**, *"Nonparametric reconstruction of the cosmic expansion with local regression smoothing and simulation extrapolation"*, Phys.Rev. D89 (2014) no.4, 043007.

In this paper we propose a new nonparametric approach which works on minimal assumptions, to reconstruct the cosmic expansion of the universe, and based on the combination of two methods:

LOESS, a locally weighted scatter-plot smoothing method; and SIMEX, a simulation-extrapolation method. My contribution consisted in performing part of the numerical calculations and contributing to the writing of the article.

My percentage contribution is estimated at about 20%.
Impact Factor: 4.643 (2014 JCR).

SB10. **Salzano V.**, Capozziello S., Napolitano N.N., Mota D.F., *"Unifying static analysis of gravitational structures with a scale-dependent scalar field gravity as an alternative to dark matter"*, Astron.Astrophys. 561 (2014) A131.

In this work we test an alternative gravity theory, inspired by the chameleon screening mechanism, and phenomenologically built on an interaction length and a coupling constant to the ordinary matter which scale with the local properties of the considered astrophysical system. We test the model with internal dynamics of clusters of galaxies, elliptical galaxies, and spiral galaxies, and find a general agreement between theory and data, with interesting correlations among the different parameters involved and the evolutionary state of the gravitational structures considered. My contribution consisted in performing all the numerical calculations; discussing the results obtained; and giving an active contribution to the article writing.

My percentage contribution is estimated at about 70%.
Impact Factor: 5.185 (2014 JCR).

SB11. **Salzano V.**, Wang Y., Lazkoz R., *"Linear dark energy equation of state revealed by supernovae?"*, Mod.Phys.Lett. A29 (2014) 1450008.

In this paper we propose a test to detect the linearity of the dark energy equation of state. The method is based on performing a chain of linear interpolations in the dark energy equation of state, with different pivot redshift values, and on checking if the inferred results are consistent or not. We conclude that current supernovae data are well described by a dark energy EoS linear in the scale factor and that there is no strong and significant evidence of any deviation from linearity. My contribution consisted in performing all the numerical calculations; discussing the results obtained; and giving an active contribution to the article writing.

My percentage contribution is estimated at about 75%.
Impact Factor: 1.198 (2014 JCR).

SB12. **Salzano V.**, Rodney S.A., Sendra I., Lazkoz R., Riess A.G., Postman M., Broadhurst T., Coe D., *"Improving Dark Energy Constraints with High Redshift Type Ia Supernovae from CANDELS and CLASH"*, Astronomy & Astrophysics 557 (2013) A64.

In this work we investigate the degree of improvement in dark energy constraints that can be achieved by extending Type Ia Supernova samples to redshifts $z > 1.5$, using prescriptions from the CANDELS and CLASH programs run with the Hubble Space Telescope. We find that with a total of 28 SN Ia at $z > 1.0$ we could improve the uncertainty in the dynamical dark energy parameter w_a from the CPL parametrization up to 21%. My contribution consisted in performing all the numerical calculations; discussing the results obtained; and giving an active contribution to the article writing.

My percentage contribution is estimated at about 55%.
Impact Factor: 5.185 (2013 JCR).

SB13. Lazkoz R., Alcaniz J., Escamilla-Rivera C., **Salzano V.**, Sendra I., *"BAO cosmography"*, JCAP 1312 (2013) 005.

In this work we explore how the BAO observed by the future survey mission *Euclid* can improve constraints on the dynamical evolution of the universe by using cosmography, a fully model-independent approach to cosmological evolution. We show that future BAO data have the potential to provide a model-independent check of the cosmic acceleration as well as a discrimination between the standard ΛCDM model and alternative mechanisms of cosmic acceleration. My contribution consisted in performing most of the numerical calculations; discussing the results obtained; and giving an active contribution to the article writing.

My percentage contribution is estimated at about 40%.
Impact Factor: 5.877 (2013 JCR).

SB14. Lazkoz R., Montiel A., **Salzano V.**, Sendra I., *"First cosmological constraints on the Superfluid Chaplygin gas model"*, Phys.Rev. D86 (2012) 103535.

In this paper we study the feasibility of a Superfluid Chaplygin gas model, which gives a unified description of the dark sector of the Universe as a Bose-Einstein condensate that behaves as dark energy while it is in the ground state and as dark matter when it is in the excited state. My contribution consisted in performing part of the numerical calculations and giving an active contribution to the article writing.

My percentage contribution is estimated at about 25%.
Impact Factor: 4.691 (2012 JCR).

SB15. Lazkoz R., **Salzano V.**, Sendra I., *"Revisiting a model-independent dark energy reconstruction method"*, Eur.Phys.J. C72 (2012) 2130.

In this paper we contribute to model-independent reconstructions methods of dark energy literature, by revisiting a model which reconstructs the dimensionless cosmological distance and its two first derivatives using a polynomial fit in different redshift windows. We update the method with new data and highlight pros and cons of the method. My contribution consisted in performing most of the numerical calculations; discussing the results obtained; and giving an active contribution to the article writing.

My percentage contribution is estimated at about 33%.
Impact Factor: 5.247 (2012 JCR).

SB16. Capozziello S., Lazkoz R., **Salzano V.**, *"Comprehensive cosmographic analysis by Markov Chain Method"*, Phys.Rev. D84 (2011) 124061.

In this work we perform a fully-detailed analysis of cosmography, a model-independent approach to the analysis of the cosmological dynamics on large scales. We study all the steps involved in its definition, from the first many theoretical assumptions, to the problems when it has to be compared with data. This work gives for the first time in literature a complete sketch about the topic and addresses how cosmography should be properly used in cosmological analysis. My contribution consisted in performing all the numerical calculations; discussing the results obtained; and giving an active contribution to the article writing.

My percentage contribution is estimated at about 75%.
Impact Factor: 4.558 (2011 JCR).

SB17. Escamilla-Rivera C., Lazkoz R., **Salzano V.**, Sendra I., *"Tension between SN and BAO: current status and future forecasts"*, JCAP 1109 (2011) 003.

In this work we study the "tension", i.e., a difference of at least 2σ level between the EoS parameters values obtained by using a certain data sets, between supernovae and baryon acoustic oscillations. We show that such tension is independent on the equation of state parametrization and on the choice of the priors, and that might be pathological in the future, when the precision of the observations will be improved so much that the tension will become more evident. My contribution consisted in performing part of the numerical calculations; discussing the results obtained; and giving an active contribution to the article writing.

My percentage contribution is estimated at about 25%.
Impact Factor: 5.723 (2011 JCR).

SB18. Mota D.F, **Salzano V.**, Capozziello S., *"Testing feasibility of scalar-tensor gravity by scale dependent mass and coupling to matter"*, Phys.Rev. D83 (2011) 084038.

In this work we consider an alternative "scalar-tensor field" gravity model defined by a Yukawa-type coupling between the field and matter and by a mass field which grows with density (chameleon-like mechanism). We analyse three different gravitational systems assumed as "cosmological indicators": type Ia supernovae, low surface brightness spiral galaxies and clusters of galaxies, finding a general agreement between theory and data. My contribution consisted in performing all the numerical calculations; discussing the results obtained; and giving an active contribution to the article writing.

My percentage contribution is estimated at about 75%.
Impact Factor: 4.558 (2011 JCR).

SB19. Lazkoz R., **Salzano V.**, Sendra I., *"Oscillations in the dark energy EoS: new MCMC lessons"*, Phys.Lett. B694 (2011) 198-208.

In this paper we study the possibility of detecting oscillating patterns in the equation of state of the dark energy using different cosmological data sets. Among the different proposals, those resulting as the "best" from the statistical analysis are compared with the standard ΛCDM using dimensionally consistent Bayesian approaches based on information criterion, and we do not find a significant evidence against dark energy oscillations. My contribution consisted in discussing the results obtained; and giving an active contribution to the article writing.

My percentage contribution is estimated at about 30%.
Impact Factor: 5.255 (2010 JCR).

SB20. Cardone V.F., Tortora C., Molinaro R., **Salzano V.**, *"The global mass - to - light ratio of SLACS lenses"*, Astronomy & Astrophysics 504 (2009) 769-788.

In this paper we study the dark matter content of early-type galaxies, focussing in particular on the presence of significant dark mass fraction within the effective radius. Using a sample of gravitational lenses, we parameterize the radial dependence of the mass-to-light ratio and we find a good agreement with the data suggesting the presence of massive dark matter haloes to explain the lensing and dynamics properties. My contribution consisted in performing part of the calculations.

My percentage contribution is estimated at about 15%.
Impact Factor: 5.185 (2009 JCR).

SB21. Capozziello S., **Salzano V.**, *"Cosmography and large scale structure by f(R) gravity: new results"*, Adv. Astron. 2009 (2009) 217420.

In this paper we review results obtained in my Ph.D. papers, Refs. [**SC1**,**SC2**], centering on how $f(R)$-gravity models can be made consistent with data, both using forecast cosmological-scale analysis (by cosmography), and using data from clusters of galaxies. My contribution consisted in writing most of the paper.

My percentage contribution is estimated at about 75%.
Impact Factor: 0.811 (2009 JCR).

5.2 Scientific publications in journals listed in Journal Citation Reports (JRC) database published before Ph.D. completion

SC1. Capozziello S., De Filippis E., **Salzano V.**, *"Modelling clusters of galaxies by f(R)-gravity"*, Mon.Not.Roy.Astron.Soc. 394 (2009) 947-959.

In this article we show the compatibility of a general class of $f(R)$-theories with clusters dynamics. My contribution consisted in: performing all the numerical calculations; discussing the results obtained; and giving an active contribution to the article writing.

My percentage contribution is estimated at about 70%.
Impact Factor: 5.103 (2009 JCR).

SC2. Capozziello S., Cardone V.F., **Salzano V.**, *"Cosmography of f(R) gravity"*, Physical Review D78 (2008) 063504.

In this work we show how to relate a model-independent cosmological approach (cosmography) to an alternative theory of gravity ($f(R)$-gravity) in order to assess possible constraints on the latter when the former is applied to cosmological data. My contribution consisted in: performing part of the required calculations and giving an active contribution to the article writing.

My percentage contribution is estimated at about 35%.
Impact Factor: 5.050 (2008 JCR).

5.3 Conference Proceedings

SD1. *"How to Reconstruct a Varying Speed of Light Signal from Baryon Acoustic Oscillations Surveys"*, **Salzano V.**, Universe 3 (2017) no.2, 35. Proceedings "Varying Constants and Fundamental Cosmology (VARCOSMOFUN'16)", Szczecin, Poland, September 11-17, 2016.

SD2. *"New tests of variability of the speed of light"*, Dąbrowski M.P., **Salzano V.**, Balcerzak A., Lazkoz R., EPJ Web Conf. 126 (2016) 04012. Proceedings "4^{th} International Conference on New Frontiers in Physics (ICNFP 2015)", Crete, Greece, August 23-30, 2015.

SD3. *"Cosmological constraints on fast transition Unified Dark Matter models"*, Lazkoz R., Leanizbarrutia I., **Salzano V.**, J.Phys.Conf.Ser. 600 (2015) no.1, 012028. Proceedings "Spanish Relativity Meeting : Almost 100 years after Einstein Revolution (ERE 2014)", Valencia, Spain, September 1-5, 2014.

5.4 Bibliometric Summary

5.4.1 Total Impact Factor

Impact Factor for the year of publication (for 2017-papers we have used 2016 numbers) and score from MNiSW according to list A from 2016.

	Impact Factor JCR	Score MNiSW
Papers entering the habilitation	30.24	225
All publications	132.057	1000

5.4.2 Citations

Number of citations at 27 October 2017.

	WoS	NASA ADS	inSpire	Google Scholar
Total Citations	359	478	477	546
Total Citations excluding self-citations	335	448	332	–
H-index	9	11	11	13

5.5 Participation in international research projects

1. CANTATA (April 2016 - present): in April 2016 I have joined the EU-funded COST (European Cooperation in Science and Technology) action *"CANTATA" - Cosmology and Astrophysics Network for Theoretical Advances and Training Actions*. Links: EU COST institutional page here; action official page here. The COST action aims to create and enforce a strongly-interconnected network of researchers working on Modified Gravity in Europe (but it is also open to non-EU countries collaborations). The project has received funds (for four years) to support scientific missions (short and long-term stays) of Ph.D. and Postdoctoral researchers who participate to the project; to organize meetings, workshops, schools for Ph.D. students and outreach activities. Within this project, I have both research and organizational duties. I am one of the four representatives of Poland; and I have been appointed as Junior Co-leader of the working group "Observational Discriminators of Modified Gravity". As such, I am in charge of coordinating the work among the members (now ~ 50) who work on this topic; I have to guarantee a constantly-maintained connection and interchange of ideas between them; to promote international collaborations among them; and I am also the official spokesperson of the group. In addition, I have to go on with my own research activity on modified gravity theories, as personal contribution for a successful yearly evaluation of the project;

2. J-PAS (January 2016 - present): I am member, by direct invitation on personal basis of the *"J-PAS - Javalambre Physics of the Accelerating Universe Astrophysical Survey"* (official webpage here). J-PAS is a planned (fully operative start in 2018) photometric sky survey which will cover ≈ 8500 square-degrees in approximately 5 years, based on the Astronomical

Observatory of Javalambre, a scientific facility located at the Sierra de Javalambre in Teruel, Spain, equipped with two telescopes. The survey will use a system of 56 narrow band filters in the optical and will be conducted by a 2.5 m telescope; the filter system was optimized to (among others) observe galaxies up to $z \sim 1.3$, emission-line galaxies up to $z \sim 2.5$, and quasars up to $z \sim 6$. Thus, with JPAS it will be possible to have insights in dark energy nature and growth of matter perturbations through BAO. I have been appointed to work within the Theory Science Working Group, where I am in charge to write and run the numerical codes to perform Fisher Matrix cosmological forecasts based on the matter power spectrum observations, given that J-PAS will be one of the most competitive photometric surveys focusing on BAO. I am also involved in a project aimed to find the best way to use the outcomes of the Fisher Matrices for some tests which could be performed by BAO (Copernican Principle, duality distance, measuring the space curvature of the universe);

3. EPI (January 2013- August 2014): I have worked first as full member and then, after ending my formal contract in 2014, as external collaborator, for the project *"EPI - Exploring the Physics of Inflation"* (official webpage here), funded directly by the Spanish Ministry of Science and Innovation through the "CONSOLIDER-Ingenio" program and involving several research groups and institutions from Spain and Europe. I have run simulations in order to establish what kind of constraints on the inflationary parameters (in particular, on the amplitude of the gravitational waves) might be expected from the "QUIJOTE - Q, U, I Joint TEnerife experiment", depending on the sensitivity of the telescopes and other architectural parameters. For this purpose, I have mainly used CosmoMC (available here) and CAMB (available here), the most common sets of routines which can calculate everything you need about primordial, radiation and matter power spectrum, and I have acquired some expertise in modifying and using both of them.

5.6 International and National Prizes for Scientific Activity

06.2017 West Pomeranian Nobel 2016 in Fundamental Science, a prestigious award conferred by the West Pomeranian Leader of Science (Zachodniopomorski Klub Liderów Nauki - ZKLN) for my work about the measurement of the speed of light on cosmological scales by using cosmic rulers and cosmic chronometers (namely, most of the scientific achievements which are the basis for this habilitation procedure). Links: Wyborcza Szczecin; Onet.pl; Nauka w Polsce; TVP3 Szczecin.

5.7 Seminars held in national and international scientific institutions

09.2017 Seminar for the "Gravitation and Cosmology" parallel session at the 44th Congress of Polish Physicists organized by Wrocław Division of the Polish Physical Society. Title: *"Varying speed of light signatures in cosmological data"*. Official conference page, here;

05.2017 Seminar for the Astrophysics Group at the National Centre for Nuclear Research (NCBJ) in Warsaw. Title: *"Varying speed of light signatures in cosmological data"*. Official announcement at the link of the group, here;

11.2016 First "CANTATA" meeting, held at the Faculty of Science of the University of Lisbon (Portugal). Presentation, in quality of co-leader, of the Working Group "Observational Discriminators", defining its role, objectives and the future moves of the group. The program of the meeting is available at the following link;

09.2016 Talk at the "Varying Constants and Fundamental Cosmology - VarCosmoFun'16" conference, organized by the University of Szczecin. Title: *"Recovering a redshift-extended VSL signal from future galaxy surveys"*. Programm available at the following link;

09.2015 Talk at the "COSMO15 - 19th International Conference on Particle Physics and Cosmology" at the University of Warsaw. Title: *"Measuring the speed of light with Baryon Acoustic Oscillations"*. Programm available at the following link;

04.2015 Talk for the Physics Seminars of the Institute of Physics at the University of Szczecin. Title: *"Measuring the speed of light with Baryon Acoustic Oscillations"*;

01.2015 "Beyond LCDM conference" organized by the Institute for Theoretical Astrophysics of the University of Oslo (Norway). Invited co-chair for the parallel discussion session: *"DE model selection and parametrisations: What do we expect to learn about DE from forthcoming data?"*;

12.2014 Talk for the Institute of Physics Seminars at the University of Warsaw. Title: *"Alternative Chameleon-inspired gravity: a phenomenological and observational approach"*. Official announcement at this link;

07.2014 Talk for the Friday Seminars of the Cosmology Group at the Institute of Physics of the University of Szczecin. Title: *"Alternative gravities vs dark energy: an observational and phenomenological approach"*;

06.2013 Talk at the "EPI conference 2013" held at the Institute of Physics of Cantabria (IFCA) in Santander (Spain). Title: *"Constraints on inflationary models with Planck and Quijote"*;

10.2008 Talk at the "VIII National Congress - INFN (National Institute of Nuclear Physics)-Iniziativa Specifica NA12" held at the University of Salerno. Title: *"Cosmography of f(R) gravity"*.

6 Teaching and divulgation

6.1 Teaching

6.1.1 University Lectures

Postdoctoral activity 2014 — *"Introduction to galaxy morphology, kinematics and dynamics and Introduction to Theoretical and Observational Cosmology"* 4 hours/1 week lecture for the "Astrophysics" course of the 4th year Physics Master at the University of the Basque Country.

Postdoctoral activity 2013 — *"Summer Scientific Campus Programm 2013"* 25 hours/1 week for the "Programa Campus cientificos de verano 2013" by the "Ministerio de Educacion, Cultura y Deporte" of Spain and the "Fundacion Espanola para la Ciencia y la Tecnologia (FECYT)".

Postdoctoral activity 2012 — *"Introduction to Cosmology"* 8 hours/2 weeks lecture for the "Gravitation and Cosmology" course of the 4th year Physics Master at the University of the Basque Country.

Postdoctoral activity 2012 — *"Introduction to Gravitational Lensing"* 2 hours/2 days lecture for the "Physics Topics" course of the 4^{th} year Physics Master at the University of the Basque Country.

6.1.2 Supervision of Students

PhD student 2016-2020 — Co-supervisor of the Ph.D. student Maria Ortiz Baños, at the University of the Basque Country. The thesis will be centered on observational tests of $f(R)$ modified gravity theories.

PhD student 2014-2018 — Co-supervisor of the Ph.D. student Iker Leanizbarrutia, at the University of the Basque Country. The thesis will be centered on observational tests for dark energy nature.

PhD student 2014-2017 — Co-supervisor of the Ph.D. student Hussain Gohar, at the Institute of Physics of the University of Szczecin. Title of the thesis: *"Thermodynamical aspects of black holes and cosmological horizons in varying fundamental constants theories"*.

Student project 2010-2011 — Co-supervisor for the end of course Thesis of the student Unai Alvarez Rodriguez, Title: *"Discovering Dark Energy: history and results"*, University of the Basque Country.

6.2 Divulgation

02.2016 — Non-technical summary of the work Ref. [**SB1**] about possible observational constraints on future cosmological singularity. Aimed for common public and published online by invitation from the on-line version of the scientific journal *New Scientist*. Available at the following link.

04.2015 Announcement of the results achieved by the work Ref. [**SA7**] on the Bulletin of the Polish Ministry of Science and Higher Education (MNiSW) on 10th April 2015. Available at the following link.

04.2015 Non-technical summary of the work Ref. [**SA7**] about the measurement of the speed of light on cosmological scales. Aimed for common public and published online, by invitation from the website *Phys.Org*, a science, research and technology portal providing the latest news on science. Available at the following link.

6.3 Organization of scientific conferences

09.2017 Member of the Organizing Committee of the scientific school "LCDM and Beyond: Cosmology Tools in Theory and in Practice. A CANTATA Cost-Action Summer School" held in Corfù (Greece) on September $4-14$. The school is part of the training actions for Ph.D. students and your Postdoctoral researchers for which the COST action CANTATA has received funding from the EU community. Link of the school, here.

09.2016 Member of the Local Organizing Committee of the scientific conference "Varying Constants and Fundamental Cosmology - VARCOSMOFUN'16" held in Szczecin on September $12-17$. Webpage of the conference here.
Guest editor for the corresponding proceeding volume, published as a special issue of the electronic open-access journal *Universe*. Link to the proceedings here.

6.4 Reviewer work

Reviewer for: (~ 20 articles) Physical Review Letters; Physical Review D; Physics Letter B; The Astronomical Journal; The European Physical Journal C; General Relativity and Gravitation; Acta Physica Polonica.